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2026 Specialist Mathematics

Year 12 Problem Solving Task

(Time allowed: 2.0 hours plus)

Problem Solving Task

Theme: Collision course

Important information: The particles in this task are considered to be dimensionless.
Time $t \in R$ is in seconds, and distance is in metres.
Position vectors $\underset{\sim}{r}$ of particles are centred at $(0, 0)$.
$\underset{\sim}{r}_A(t)$ is the position of particle A and $\underset{\sim}{r}_B(t)$ is the position of particle B at time t.
The position of A relative to B (i.e. the position of A as seen by B) is given by $\underset{\sim}{r}_{AB} = \underset{\sim}{r}_A - \underset{\sim}{r}_B$.
$\underset{\sim}{v}_A(t)$ is the velocity of particle A and $\underset{\sim}{v}_B(t)$ is the velocity of particle B at time t.
The velocity of A relative to B (i.e. the velocity of A as seen by B) is given by $\underset{\sim}{v}_{AB} = \underset{\sim}{v}_A - \underset{\sim}{v}_B$.
Two particles are said to **collide** at time t when their relative position is a zero vector and their relative velocity is a nonzero vector. The collision is **head on** when $\underset{\sim}{v}_A(t)$ and $\underset{\sim}{v}_B(t)$ are opposite in direction.
The particles disintegrate after a collision.
Two particles are said to **meet** at time t when their relative position is a zero vector and their relative velocity is a zero vector. The particles move on after they meet.

Assumed knowledge: Parametric equations; position and velocity vectors in i and j form; scalar products; vector calculus; loci of particles in Cartesian form; ellipses; length of a curve; parallel and orthogonal vectors; transformations.

Part I (70 minutes plus)

The position vectors of particle A and particle B are respectively $\underset{\sim}{r}_A = (t+1)\underset{\sim}{i} + e^t \underset{\sim}{j}$ and $\underset{\sim}{r}_B = e^t \underset{\sim}{i} + (t+1)\underset{\sim}{j}$ for $t \in R$.

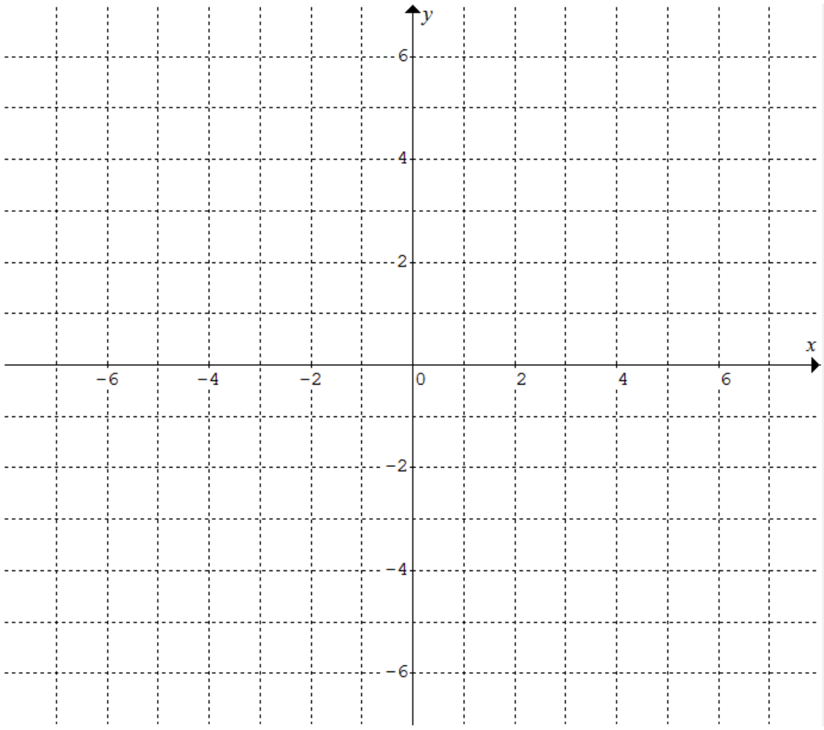
a. Determine the position and velocity of particle B relative to particle A at time t.

b. Show that $\frac{d}{dt}\underset{\sim}{r}_{BA} = \underset{\sim}{v}_{BA}$.

c. Show that particle A and particle B **meet** at position $(1,1)$ when $t = 0$.

d. Determine the speed and direction of motion of each particle when they meet.

e. Find the Cartesian equations of the loci of particle A and particle B. Sketch both loci on the same axes below. Label each with A or B. Use an arrow head to show the direction of motion of each.



Consider the following position vectors for the two particles.
$\tilde{r}_A = (t+\alpha)\tilde{i} + e^t \tilde{j}$ and $\tilde{r}_B = e^t \tilde{i} + (t+\alpha)\tilde{j}$ for $t \in R$, where constant $\alpha \in R^+$.

f. Discuss the effects of increasing α on the loci of particle A and particle B.

g. Find the value(s) of α for the particles to **collide**. In terms of α, find t, $\tilde{r}_A$ and $\tilde{r}_{AB}$ when the two particles collide.

h. Show that the particles will not collide at right angle.

Now consider the following position vectors for the two particles.
$\tilde{r}_A = (-t+\beta)\tilde{i} + e^{-t}\tilde{j}$ and $\tilde{r}_B = e^{t}\tilde{i} + (t+\beta)\tilde{j}$ for $t \in R$, where constant $\beta \in R^{+}$.

i. Compare the motion of particle A now ($\tilde{r}_A = (-t+\beta)\tilde{i} + e^{-t}\tilde{j}$) with its motion previously ($\tilde{r}_A = (t+\alpha)\tilde{i} + e^{t}\tilde{j}$).

j. Find the value(s) of β such that the paths of the two particles intersect.

k. Find the value(s) of β such that the two particles will collide.

l. State the type of collision of the two particles in part k. Verify your answer.

End of Part I

Information given in Part I:

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$\underset{\sim}{v}_A(t)$ is the velocity of particle A and $\underset{\sim}{v}_B(t)$ is the velocity of particle B at time t.
The velocity of A relative to B (i.e. the velocity of A as seen by B) is given by $\underset{\sim}{v}_{AB} = \underset{\sim}{v}_A - \underset{\sim}{v}_B$.
Two particles are said to **collide** at time t when their relative position is a zero vector and their relative velocity is a nonzero vector. The collision is **head on** when $\underset{\sim}{v}_A(t)$ and $\underset{\sim}{v}_B(t)$ are opposite in direction.
The particles disintegrate after a collision.
Two particles are said to **meet** at time t when their relative position is a zero vector and their relative velocity is a zero vector. The particles move on after they meet.

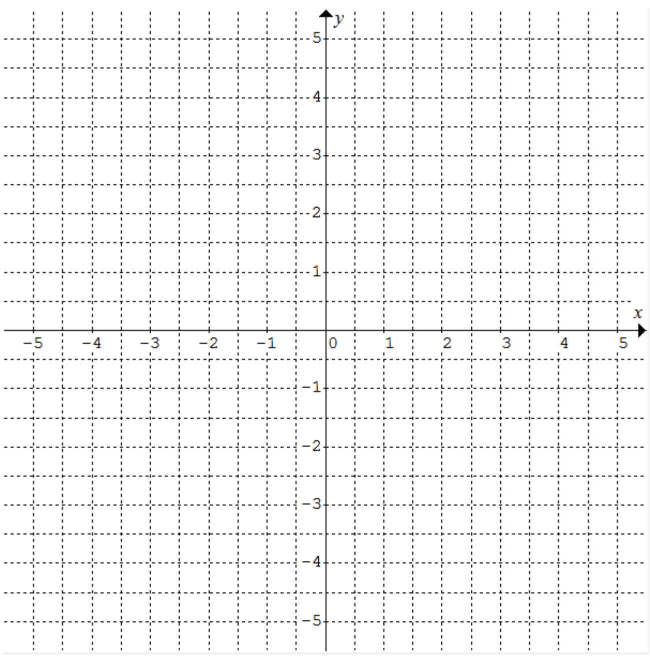
In Part II the particles are particle C and particle D.

Part II (70 minutes plus)

Particle C and particle D have position vectors $\underset{\sim}{r}_C = -2\cos(t)\,\underset{\sim}{i} + \sin(t)\,\underset{\sim}{j}$ and $\underset{\sim}{r}_D = 2\left(\sqrt{2} + \cos(t)\right)\underset{\sim}{i} + \sin(t)\,\underset{\sim}{j}$ respectively for $t \geq 0$.

a. Determine the Cartesian equations of the loci of particle C and particle D.

b. Sketch both loci (dotted curves) on the same axes below assuming that there is no collision. Label each with C or D. Use an arrow head to show the direction of motion of each. State the starting positions (at $t = 0$) of the particles.



c. Determine the time and position when the particles **collide**.

d. Determine the maximum distance travelled by each particle.

e. Use solid curves (over the dotted curves) in the graph in part b to show the paths of the particles leading to collision.

f. Show that the collision of the two particles occurs at right angle.

Head on collision of the particles can occur under the following two conditions.
(1) The motion of particle C remains the same.
(2) The path of D is different. It is the transformation (two translations) of the path of C. Note: There are different possibilities for you to decide.

g. Write down a possible position vector $\tilde{r}_D$ for head on collision of the two particles to occur.

End of Part II