Math Lesson (Suitable for Years 9 to 12)
Decimal places and significant figures in measurements

The **precision** of a measurement or an instrument is the smallest unit with which the measurement can be made.

Consider measuring the length of an A4 sheet of paper with a ruler which is graduated in centimetres and millimetres. The measurement is 29 cm 7 mm. This measurement can be expressed as 297 mm, 29.7 cm, 0.297 m or 0.000297 km.
They are equivalent forms with different number of decimal places and units, and they all have the same precision of 1 mm.

Consider estimating the size of the crowd at a football match (in terms of thousands of people) to be 17 000 people. This estimation has a precision of 1 000 people.

The precision of a decimal measurement is given by the place value of the last decimal place together with the unit.
For a measurement in whole number with zeros at the end, the precision is indeterminable unless the instrument used is known.

The **accuracy** of a measurement depends on the instrument and the way it is used. It is given by the number of figures that the instrument allows for the measurement. They are called **significant figures**. The last significant figure in the measurement corresponds to the precision of the instrument.

Referring to the two examples above, the length of the A4 sheet has 3 significant figures and therefore the accuracy of the measurement is 3, whilst the estimation of the football crowd has 2 significant figures and its accuracy is 2.

Example 1 A trip-counter in a car shows a distance of 538.0 km. Express the distance in metres.
What is the precision of the trip-counter?
What is the accuracy of the measurement?

The distance is 538 000 m.
The precision is 0.1 km, or 100 m.
Irrespective of the unit that the measurement is expressed in, the accuracy is 4 because the measurement has 4 significant figures in both cases. The last significant figure in a measurement indicates the precision of the measurement.

This example highlights the difficulty in stating the accuracy (number of significant figures) of a measurement if its precision is not explicitly given.

Given 538 000 m only, one cannot say how many significant figures it has. Is it 3, 4, 5 or 6?

This difficulty can be overcome by expressing measurements with known precision in **standard form** as shown below.

$17000 \text{ people} = 1.7\times10^4 \text{ people}$ (2 sig. fig.)

$297 \text{ mm} = 2.97\times10^2 \text{ mm}$ (3 sig. fig.)
$29.7 \text{ cm} = 2.97\times10^1 \text{ cm}$ (3 sig. fig.)
$0.297 \text{ m} = 2.97\times10^{-1} \text{ m}$ (3 sig. fig.)
$0.000297 \text{ km} = 2.97\times10^4 \text{ km}$ (3 sig. fig.)

$538.0 \text{ km} = 5.380\times10^2 \text{ km}$ (4 sig. fig.)
$538000 \text{ m} = 5.380\times10^5 \text{ m}$ (4 sig. fig.)

A measurement in standard form has a decimal number (between 1 and 10) multiplied by a power of 10.
In standard form the last figure in the decimal number, together with the power of 10 and the unit, gives the precision, whilst the number of figures in the decimal number indicates the accuracy.

Is a precise measurement an accurate measurement?

Example 1 Two lengths are measured with different instruments to be 525.0 km and 0.525 m. Compare the two measurements in terms of accuracy and precision.

Measurement	Accuracy	Precision
525.0 km	4	0.1 km (100 m)
0.525 m	3	0.001 m

The first measurement is more accurate because it has more significant figures.
The second measurement is more precise because it is measured to the nearest 0.001 m in comparison with nearest 100 m in the first.

To further compare the accuracy of the measurements, calculate the % uncertainty (error) of each one.

First: $\dfrac{0.1\,\text{km}}{525.0\,\text{km}}\times100\% \approx 0.02\%$

Second: $\dfrac{0.001\,\text{m}}{0.525\,\text{m}}\times100\% \approx 0.2\%$

Clearly, the first has a lower % uncertainty, .: it is more accurate. The % uncertainty decreases as the number of significant figures increases.

itute.com

How many significant figures are there if the precision is not implicit in the measurement?

Example 1

A ship leaves port and sails 130.8 km NW and then 200 km $N 52° W$.

For 130.8 km the precision is implied by the measurement, .: it has 4 significant figures.

For 200 km and $52°$, the precision of each measurement is not implicit. They are usually taken as 100% accurate. The number of significant figures in each case is therefore infinite and the precision is undefined.

Exercise

Q1 Find the precision of the following measurements.
105.2 m, 604 km, 1.030 g, 0.005 L, 3.0006 kg

Q2 Find the precision of the following measurements.
A line above a zero indicates the place where precision lies.
20 cm, $5\overline{00}0$ w, $37\overline{0}00$ kg, $350\overline{0}$ L

Q3 Express the measurements in Q1 and Q2 in standard form if possible. State the number of significant figures in each case.

Q4 Consider the following measurements.
12.23 g, 3.0 mg, 0.5 kg, 0.020010 kg
Which one is
(a) the most accurate?
(b) the most precise?
(c) the least accurate?
(d) the least precise?

Q5 Calculate the % uncertainty of each measurement in Q4.

Q6 A water tank has the following measurements:
Height 130.0 cm, total surface area 21000 cm^2 and volume 95.0 L
What is the accuracy of each measurement?