**Math Lesson (Suitable for Years 7 to 10)**
**Ratios, proportions, scales and scale factors**

A **ratio** is a mathematical expression consisting usually of two whole numbers, e.g. 2 : 3 (read 2 to 3). It is used to compare two quantities. 2 : 3 means that for every two parts of the first quantity there are three parts of the second, i.e. the first quantity is $\frac{2}{3}$ of the second quantity.

Example 1 In a wardrobe there are 10 shirts and 5 coats. The ratio of shirts to coats is 10 : 5. Dividing both numbers by 5 to give 2 : 1 which is equivalent to and simpler than 10 : 5.

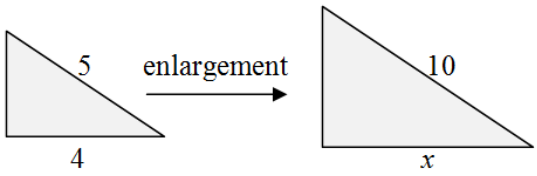
Example 2 Concrete is obtained by mixing 1 part of cement, 4 parts of sand and 5 parts of gravel. There are 10 parts of concrete.
The ratio of cement to concrete is 1 : 10, i.e. the amount of cement required is $\frac{1}{10}$ of the amount of concrete formed. The ratio of sand to concrete is 4 : 10 that can be simpified to 2 : 5, i.e. the amount of sand required is $\frac{2}{5}$ of the amount of concrete.

Example 3 A small fortune $70,000 is to be divided between two brothers according to the ratio of their ages which is 16 : 19.
The sum of their ages is 35. The ratio of the age of the younger to the sum of the ages is 16 : 35, $\therefore$ the younger brother receives $\frac{16}{35}$ of $70,000,
i.e. $\frac{16}{35} \times 70{,}000 = \$32{,}000$. Hence the elder brother receives $70,000 – $32,000 = $38,000.

Two quantities are in **proportion** if their ratio remains the same when they vary. We say one quantity is directly proportional to the other quantity.

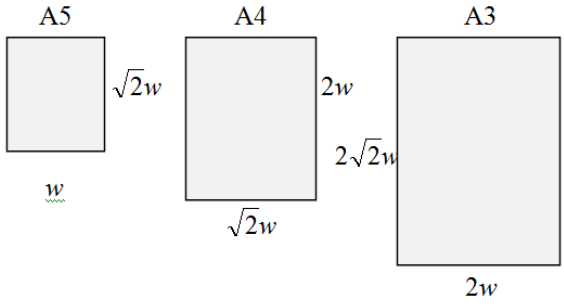
Example 4 Refer to example 2. To make concrete the ratio of sand to gravel is 4 : 5. To make 10 buckets of concrete, 4 buckets of sand and 5 buckets of gravel are required. To make 20 buckets of concrete, 8 buckets of sand and 10 buckets of gravel are required. It is the same ratio 4 : 5 of sand to gravel. The amount of sand and the amount of gravel are in proportion, i.e. the amount of sand is directly proportional to the amount of gravel in making concrete.

Example 5 When a 2-dimensional figure is enlarged or reduced, the side lengths are in proportion, i.e. the ratio of the side lengths remains constant. When one side length is doubled, the other side length is also doubled to give the same ratio.




To find $x$ use the idea that the two sides of each triangle are in the same proportion, .: the ratio of the side lengths remains constant for the two triangles.
$5:4=8:x \ \therefore \frac{5}{4}=\frac{8}{x}, \ 5x=32, \ x=6.4.$

Example 6 Copying papers have many sizes, e.g. A5, A4, A3 etc. The ratio of length : width is the same value 1.4140, i.e. $\sqrt{2}$ for these sizes. The length and the width are in proportion.




For a map (or a scale drawing) the ratio of length measure on the map to length measure of the actual locality is called the **scale** of the map.
For example, 1 : 20,000 appears on a map. It means 1 cm on the map represents 20,000 cm (200 m) in real distance.

The length measure on the map is $\frac{1}{20{,}000}$ of the real distance.

$\frac{1}{20{,}000}$ is called the **scale factor** of the map or drawing.
In this case the scale factor is less than one and the map represents a reduction of the real thing.

*In determining the scale factor the ratio of length measure in the drawing to the length measure of the real distance is considered (not vice versa).*

Example 7
1cm on a map represents 5 km in real distance.
Find the scale factor used in drawing the map.

1 cm represents 500 000 cm

$\therefore$ the scale is 1 : 500 000, the scale factor is $\dfrac{1}{500{,}000}$

Example 8 The length of an ant in a scale drawing is 250 times the actual length. What is the scale of the drawing?
The length of the ant in the drawing is 0.5 m, what is the actual length of the ant?

The scale factor is 250, i.e. it is an enlargement.
Length in drawing : length in real life = 250 : 1.
The scale of the drawing is 250 : 1.

Let $x$ m be the real length of the ant.
Use the idea of proportion, $250 : 1 = 0.5 : x$
$\therefore \dfrac{250}{1} = \dfrac{0.5}{x}, \therefore\ x = 0.002$
The real length of the ant is 0.002 m or 2 mm.

Example 9 In a large poster the length of a 2.8 m long garden bench is 3.5 m. What is the scale factor of the poster?

The scale used in the poster is:
length of bench in poster : real length = 3.5 : 2.8
Divide both numbers by 2.8 to obtain 1.25 : 1
$\therefore$ the scale factor is 1.25

**Exercise**

(1) Change the following ratios to simplest form.
a. 6 : 14 b. 1.3 : 0.7 c. $\dfrac{2}{3} : 4$ d. $1\dfrac{1}{3} : 2\dfrac{1}{2}$

(2) Concrete is obtained by mixing 1 part of cement, 4 parts of sand and 5 parts of gravel. Write down the ratio of gravel to cement, and the ratio of gravel to sand. How many buckets of sand are required to mix with 8 buckets of gravel to make cement?

(3) Divide \$28,000 into two amounts in the ratio of 2 : 3.

(4) Find the value of $x$.
a. $x:5=10:35$ b. $27:45=9:x$ c. $(x+1):5=x:3$

(5) The scale shown on a map is 1 : 200,000. Find the real distance apart in km between two points if the two points on the map are 4 cm apart.

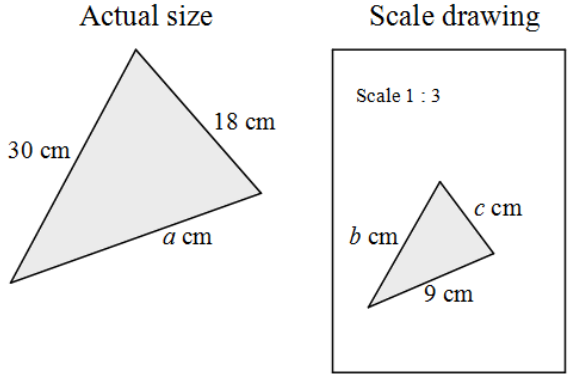
(6) The width of a A5 sheet is 14.9 cm.
Find the width and the length of a A4 sheet.

(7) 1cm on a map represents 2.5 km in real distance.
Find the scale factor used in drawing the map.

(8) The length of the stem of a flower in a scale drawing is 2.5 times the actual length.
What is the scale of the drawing?
The length of the stem in the drawing is 0.5 m.
What is the actual length of the stem?

(9) In a large poster the length of a 3.5 m long ladder is 4.9 m.
What is the scale factor of the poster?

(10) Consider the following:




Find the values of $a$, $b$ and $c$.