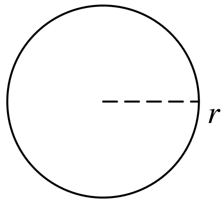
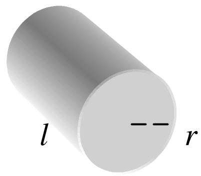
Math Lesson (Suitable for Years 8 to 10)
Areas and volumes of regular and composite solids

Surface area formulae

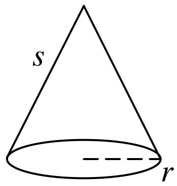
Sphere: $SA = 4\pi r^2$
(SA: surface area)



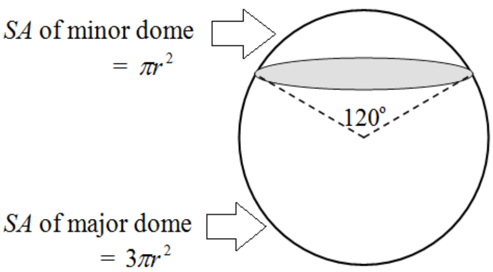
Cylinder:
Two ends + curved surface
$TSA = 2\pi r^2 + 2\pi rl$
$= 2\pi r(r + l)$
(TSA: total surface area)



Cone: Base + curved surface
$TSA = \pi r^2 + \pi rs$



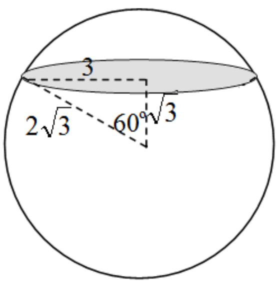
Do you know the following facts about a sphere of radius r?



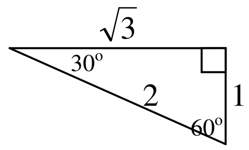
The above is true only when the minor dome subtends an angle of 120^o at the centre of the sphere.

The following examples involve the use of these two particular domes, a cylinder and a cone to form composite solids.

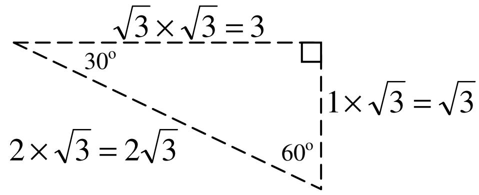
Preliminary: Consider a sphere of radius $2\sqrt{3}$ units. It is divided into a major dome and a minor dome such that the minor dome subtends an angle of 120^o at the centre of the sphere.



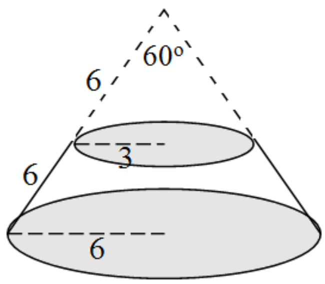
The triangle in the above drawing is similar to the special 30-60-90 triangle shown below. It is enlarged by a scale factor of $\sqrt{3}$.



Multiply each side of the 30-60-90 triangle by $\sqrt{3}$

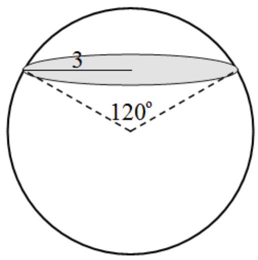


Example 1 Find the area of the curved surface of the frustum of the cone. (Measurements in cm)



Area = πrs (large cone) – πrs (small cone)
$= \pi(6)(12) - \pi(3)(6) = 90\pi$ cm^2

Example 2 Find the surface areas of the minor dome and the major dome. (Measurements in cm)

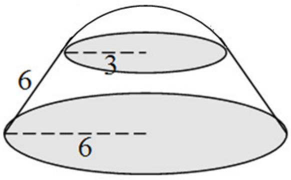


The radius of the sphere is $2\sqrt{3}$ cm (refer to preliminary).

Area of minor dome = $\pi r^2 = \pi\left(2\sqrt{3}\right)^2 = 12\pi$ cm^2

Area of major dome = $3\pi r^2 = 3\pi\left(2\sqrt{3}\right)^2 = 36\pi$ cm^2

Example 3 Find the *TSA* of the composite solid formed by placing the minor dome in Example 2 on top of the frustum of the cone in Example 1.



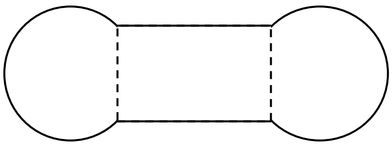
TSA

= minor dome + frustum of cone + base

$= 12\pi + 90\pi + \pi(6)^2$

$= 138\pi \text{ cm}^2$

Example 4 Find the *TSA* of the composite solid formed by placing a major dome discussed in Example 2 on each end of a 10 cm long cylinder.



TSA

= 2 major domes + cylindrical curved surface

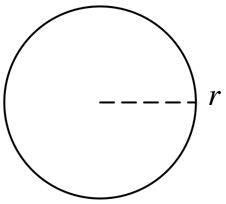
$= 2\times 36\pi + 2\pi rl$

$= 2\times 36\pi + 2\pi(3)(10)$

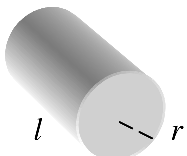
$= 132\pi \text{ cm}^2$

Volume formulae

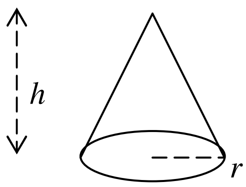
Sphere: $V = \frac{4}{3}\pi r^3$



Cylinder: $V = \pi r^2 l$



Cone: $V = \frac{1}{3}\pi r^2 h$



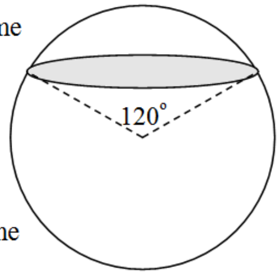
Do you know the following facts about a sphere of radius *r*?

Volume of minor dome

$\frac{5}{32}\times\left(\frac{4}{3}\pi r^3\right)$

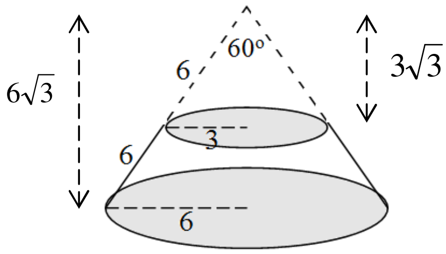
Volume of major dome

$\frac{27}{32}\times\left(\frac{4}{3}\pi r^3\right)$



The above is true only when the minor dome subtends an angle of 120° at the centre of the sphere.

Example 5 Find the volume of the frustum of the cone. (Measurements are in cm)



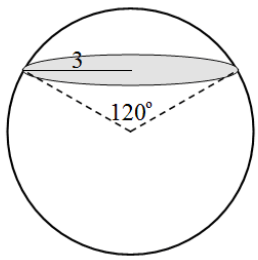
The heights of the cones were obtained by the method outlined in the preliminary, using scale factors 3 and 6.

Volume

= large cone – small cone

$= \frac{1}{3}\pi(6)^2\left(6\sqrt{3}\right) - \frac{1}{3}\pi(3)^2\left(3\sqrt{3}\right) = 63\sqrt{3}\pi \text{ cm}^3$

Example 6 Find the volumes of the minor dome and the major dome. (Measurements are in cm)



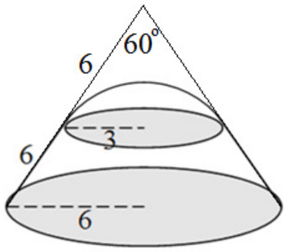
Minor dome:

$V = \frac{5}{32}\times\left(\frac{4}{3}\pi r^3\right) = \frac{5}{24}\times\pi\left(2\sqrt{3}\right)^3 = 5\sqrt{3}\pi \text{ cm}^3$

Major dome:

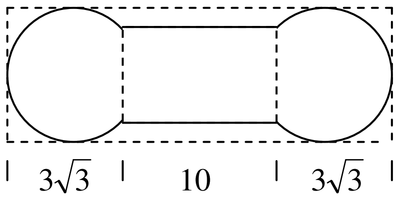
$V = \frac{27}{32}\times\left(\frac{4}{3}\pi r^3\right) = \frac{9}{8}\times\pi\left(2\sqrt{3}\right)^3 = 27\sqrt{3}\pi \text{ cm}^3$

Example 7 Find the volume of material to be cut from an appropriate cone to form the composite solid discussed in Example 3.



Volume of material to be removed
= volume of small cone – volume of minor dome
$= \frac{1}{3}\pi(3)^2\left(3\sqrt{3}\right) - 5\sqrt{3}\pi = 4\sqrt{3}\pi$ cm^3

Example 8 Find the volume of material to be cut from an appropriate cylinder to form the composite solid discussed in example 4.



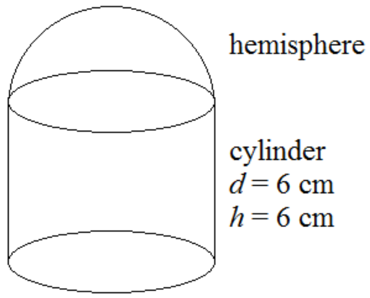
Radius of the large cylinder = $2\sqrt{3}$ cm
Length of the large cylinder = $6\sqrt{3} + 10$ cm

Volume of material to be removed
= Large cylinder – small cylinder – 2 major domes
$= \pi\left(2\sqrt{3}\right)^2\left(6\sqrt{3} + 10\right) - \pi(3)^2(10) - 2\left(27\sqrt{3}\pi\right)$
$= 6\pi\left(3\sqrt{3} + 5\right)$ cm^3

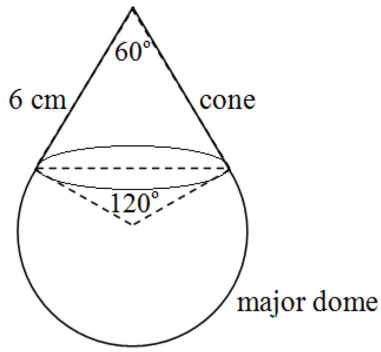
Exercise

Find the *TSA* and volume of each of the following three-dimensional composite solids.

(1)



(2)



(3)