# Math lesson (suitable for Years 8 to 12)

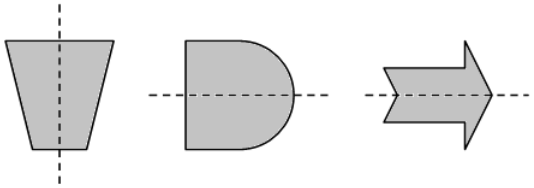
**Symmetry in 2D figures under reflection, rotation or translation** © itute 2018

**Symmetry** means exact likeness in size and shape.

### Symmetry under reflection

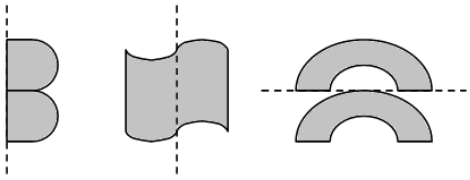
Some 2D figures remain the same in all respects after reflection in one or more lines. Such a line is called **axis (or line) of symmetry**.
We say the figures are symmetrical about their axes of symmetry.

Example 1 The following 2D figures show symmetry under reflection in the dotted lines. The dotted lines are axes of symmetry.

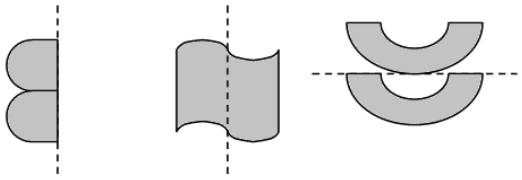


Example 2 The following 2D figures do not show symmetry under reflection in the dotted lines.

Before reflection



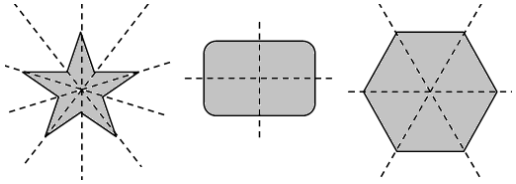
After reflection



After reflection the resulting figure is different from the original figure in each case. They do not show symmetry under reflection. The dotted lines are not axes of symmetry.

Some 2-D figures may have more than one axes of symmetry.

Example 3 The dotted lines are some of the axes of symmetry for the following figures.



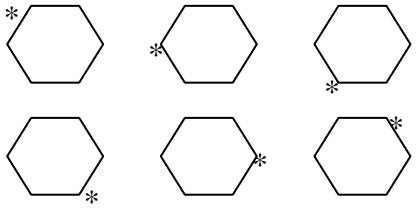
In the last figure there are three more axes of symmetry. Draw them.

### Symmetry under rotation

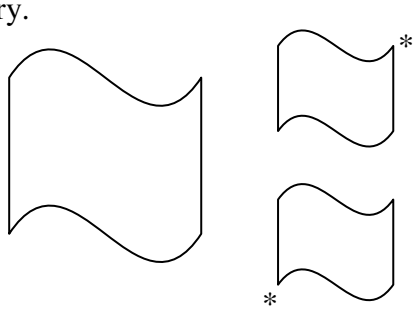
If a 2-D figure has more than one axes of symmetry, they intersect each other at the same point which is the 'centre' of the figure.

The angles made by these axes of symmetry are always equal.

If such a figure is rotated about its centre through certain angle (double the angle made by two adjacent axes of symmetry), it appears the same as before. It has rotational symmetry.
e.g. a regular hexagon has 6 axes of symmetry and it has **6-fold rotational symmetry**.

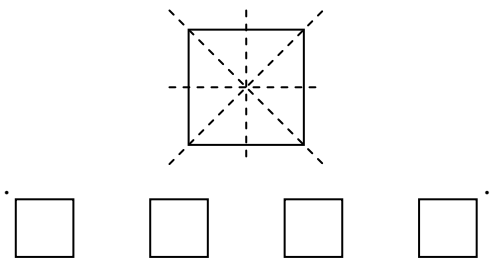


The number of axes of symmetry indicates the number of rotations giving the same appearance of the figure. However, the converse is not necessary true, e.g. the shape shown below has
2-fold rotational symmetry but it has no axis of symmetry.



Example 1 A square has n-fold rotational symmetry. What number is n? Draw diagrams to show the n-fold symmetry.

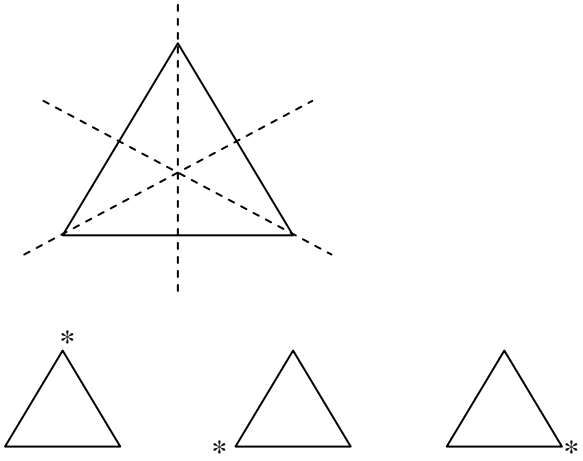
Solution: n = 4 because 4 axes of symmetry can be drawn through the same point and they are separated by the same angle.



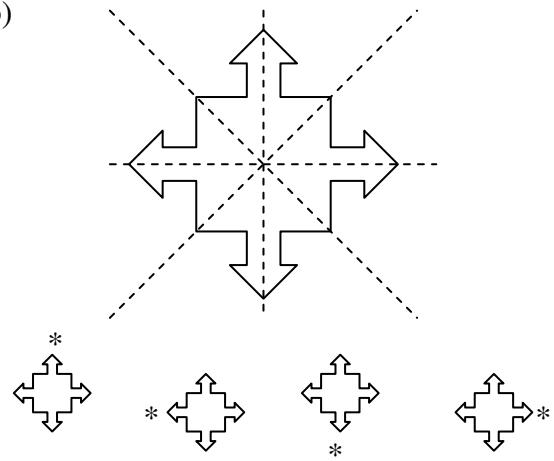
Example 2 Give an example of a 2-D figure that has (a) 3-fold rotational symmetry (b) 4-fold rotational symmetry. Illustrate with diagrams.

Solutions:

(a)



(b)



*Symmetry under translation*

Some 2-D figures or patterns exhibit symmetry (i.e. remain the same) under translations.

Example 1 The following pattern extends to infinity horizontally and vertically.

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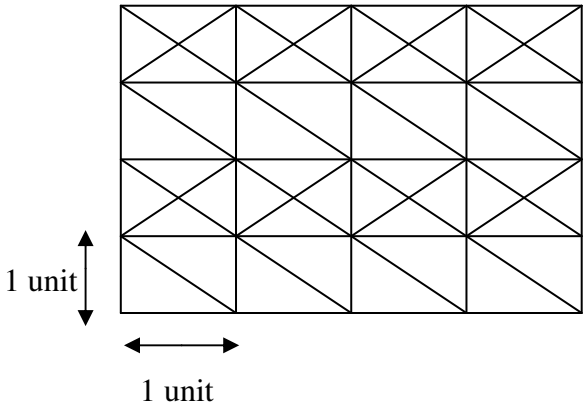
This pattern exhibits symmetry under horizontal translation by 2 units and symmetry under vertical translation by 1 unit.

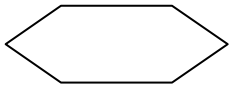
Example 2 The following design is an example of **tessellations**. A tessellation is a pattern formed by shapes covering an entire area without leaving gaps and/or overlapping.



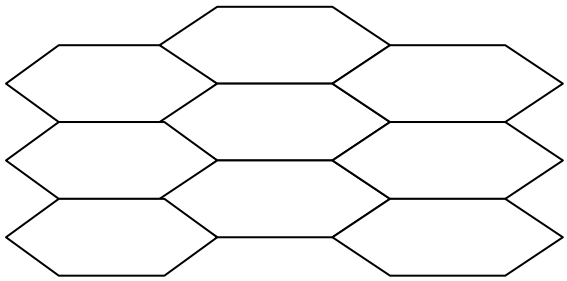

Describe the symmetry of the above design under translations.

Solution: Symmetry under horizontal translation by 1 unit. Symmetry under vertical translation by 2 units.

Example 3 Design a tessellation based on the composite 2-D figure shown below. Is it possible to design a different pattern using the same figure?



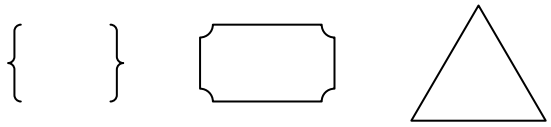
Solution:



Not possible.

**Exercise**

1) For the following 2-D figures identify those that have symmetry under reflection. Show axes of symmetry.



2) A regular pentagon has n-fold rotational symmetry. What number is n? Draw diagrams to show the n-fold symmetry.

3) Give an example of a 2-D figure that has 2-fold rotational symmetry. Illustrate the rotational symmetry with diagrams.

4) Give an example of a 2-D figure that has 3-fold rotational symmetry. Illustrate the rotational symmetry with diagrams.

5) Design a 2-D figure that has 3-fold rotational symmetry but no axes of symmetry.

6) What type of symmetry is displayed by the following designs?

(a)

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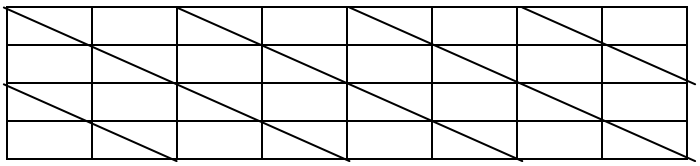
(b)



7) Explain what a tessellation is. Are the designs in question 5 tessellations? Explain.

8) Design a tessellation using a combination of the two shapes.