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Math Lesson (Suitable for Years 8 to 10) Transposition & Reasoning

A mathematical formula is a relationship among quantities. Usually it is written in the format that one quantity is expressed in terms of the other quantities, i.e. the quantity is the **subject** of the formula.

Some formulas that you have learnt are:

(a) $C = 2\pi r$ (b) $A = \pi r^2$ (c) $V = \frac{4}{3}\pi r^3$

(d) $A = 4\pi r^2$ (e) $A = \frac{1}{2}(a+b)h$ (f) $V = \frac{1}{3}\pi r^2 h$

(g) $A = 2\pi r(r+s)$ (h) $A = \sqrt{s(s-a)(s-b)(s-c)}$

The process in changing the subject of a formula is called **transposition**. It involves the transfer of terms or factors from one side of the equal sign to the other. *A term remains a term and a factor remains a factor after each transfer.*

Example 1 Transpose formulas (a), (b), (c), (d) and (f) to make *r* the subject in each case.

$C = 2\pi r$, $r = \frac{C}{2\pi}$

$A = \pi r^2$, $r^2 = \frac{A}{\pi}$, $r = \sqrt{\frac{A}{\pi}}$

$V = \frac{4}{3}\pi r^3$, $r^3 = \frac{3V}{4\pi}$, $r = \sqrt[3]{\frac{3V}{4\pi}}$

$A = 4\pi r^2$, $r^2 = \frac{A}{4\pi}$, $r = \sqrt{\frac{A}{4\pi}}$, $r = \frac{1}{2}\sqrt{\frac{A}{\pi}}$

$V = \frac{1}{3}\pi r^2 h$, $r^2 = \frac{3V}{\pi h}$, $r = \sqrt{\frac{3V}{\pi h}}$

Example 2 Transpose formula (e) to make *b* the subject.

$A = \frac{1}{2}(a+b)h$, $a+b = \frac{2A}{h}$, $b = \frac{2A}{h} - a$

Example 3 For the formula $A = 2\pi r(r+s)$, express *s* in terms of variables *A* and *r*.

$A = 2\pi r(r+s)$, $r+s = \frac{A}{2\pi r}$, $s = \frac{A}{2\pi r} - r$

Example 4 Transpose formula (h) to make $s-b$ the subject.

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s(s-a)(s-b)(s-c) = A^2$, $s-b = \frac{A^2}{s(s-a)(s-c)}$

Transposition of inequalities

Example 1 Show that if $a \ge b$, then $a-b \ge 0$.

$a \ge b$, transpose *b* to the left side to give $a-b \ge 0$.

Example 2 Show that if $\frac{y}{x} \ge 1$, then $\frac{x}{y} \le 1$.

$\frac{y}{x} \ge 1$, take the reciprocal of both sides, this requires the reversal of the inequality sign, $\frac{x}{y} \le 1$.

Example 3 Show that if $x > \frac{1}{2}$ and $y > \frac{1}{2}$, then $\frac{1}{x+y} < 1$.

Since $x > \frac{1}{2}$ and $y > \frac{1}{2}$, then $x+y > \frac{1}{2} + \frac{1}{2}$,

$\therefore x+y > 1$.

Take the reciprocal of both sides to obtain $\frac{1}{x+y} < 1$.

(Note: the inequality sign is reversed).

Example 4 Show that if $\frac{y}{x-1} < 1$ and $x > 1$, then $x-y > 1$.

$\frac{y}{x-1} < 1$, since $x > 1$, $\therefore x-1$ is positive.

Transpose $x-1$ to the right side to give $y < x-1$ which is equivalent to $x-1 > y$. Transpose 1 and *y* to obtain $x-y > 1$.

Example 5 Show that if $\frac{y}{x-1} < 1$ and $x < 1$, then $x-y < 1$.

Since $x < 1$, $\therefore x-1$ is negative.

Transpose $x-1$ to the right side and reverse the inequality sign because $x-1$ is negative.

Now the inequality becomes $y > x-1$ or $x-1 < y$.

Transpose 1 and *y* to obtain $x-y < 1$.

Example 6 Show that $a^2+b^2 \geq 2ab$ always.

A perfect square is always positive, $\therefore (a-b)^2 \geq 0$
Expand the left side, $a^2-2ab+b^2 \geq 0$. Transpose $2ab$ to obtain $a^2+b^2 \geq 2ab$.

Example 7 Show that if $a>0$ and $b>0$, then $\frac{a+b}{2} \geq \sqrt{ab}$.

Start from $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$.
Expand the left side, $\left(\sqrt{a}\right)^2 - 2\sqrt{a}\sqrt{b} + \left(\sqrt{b}\right)^2 \geq 0$, simplify to $a-2\sqrt{ab}+b \geq 0$.
Transpose $2\sqrt{ab}$ to the right side, $a+b \geq 2\sqrt{ab}$ and then 2 to the left to obtain $\frac{a+b}{2} \geq \sqrt{ab}$.

Exercise

(1) Transpose each of the following formulas to make the pro-numeral in [ ] the subject.

(a) $I = PRT$ [R]
(b) $A = \frac{1}{2}bh$ [h]
(c) $A = 2\pi r(r+s)$ [s]
(d) $V = \frac{1}{3}\pi r^2 h$ [h]
(e) $R = \frac{V}{I}$ [I]
(f) $s = \frac{1}{2}(u+v)t$ [t]
(g) $v = u + at$ [a]
(h) $y = \sqrt{1-x^2}$ [x]

(2) Show that if $a \leq b$, then $a-b \leq 0$.

(3) Show that if $x>1$ and $y>1$, then $\frac{1}{x+y} < \frac{1}{2}$.

(4) Show that if $\frac{y-1}{x} < 1$ and $x>0$, then $y-x<1$.

(5) Show that if $\frac{y-1}{x} < 1$ and $x<0$, then $y-x>1$.

(6) Show that if *a* and *b* are both positive, or both negative values, then $\frac{a}{b}+\frac{b}{a} \geq 2$.

(7) Prove that if *a* and *b* are integers of opposite signs, then $\frac{a}{b}+\frac{b}{a} \leq -2$.

(8) Prove that $x+4 \geq 4\sqrt{x}$ for $x>0$.