2004 VCAA Further Math Exam 1 Solutions

SECTION A
Core

1	2	3	4	5	6	7	8	9	10
B	B	C	D	C	C	A	A	C	B

11	12	13
E	A	D

Q1 There are 26 entries in the leaf, $\therefore$ B

Q2 $Q_L = 30$, $Q_U = 43$, $IQR = 43 - 30 = 13$ $\therefore$ B

Q3 95 is 10 (i.e. 2σ) higher than 85 (the mean). The percentage within $\pm 2\sigma$ of the mean is 95%, $\therefore$ 5% is outside $\pm 2\sigma$ of the mean and hence 2.5% is greater than $\mu + 2\sigma$, i.e. 95. $\therefore$ C

Q4 $\dfrac{9+3+1+0+1}{20} \times 100\% = 70\%$ $\therefore$ D

Q5 $\dfrac{0\times 6 + 1\times 9 + 2\times 3 + 3\times 1 + 4\times 0 + 5\times 1}{20} = 1.15$ $\therefore$ C

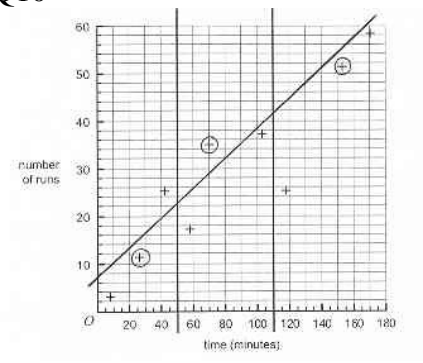
Q6 $44 + 9 + 8 = 61$ $\therefore$ C

Q7 The 11-12 students who sometimes used
$= 73 - 47 - 8 = 18$
The 7-10 students who sometimes used
$= 58 - 16 - 18 = 24$
Percentage of 7-10 students who sometimes used
$= \dfrac{24}{217} \times 100\% = 11.1\%$ $\therefore$ A

Q8 Slope $\approx -\dfrac{210}{19} \approx -11$,
vertical axis intercept ≈ 210 $\therefore$ A

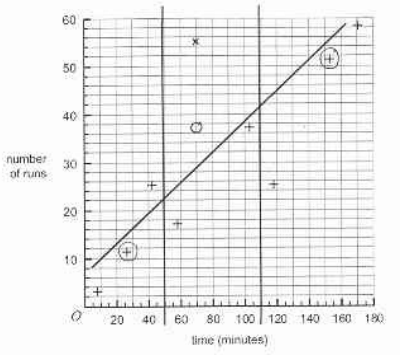
Q9 Coefficient of determination
$= r^2 = (-0.9260)^2 = 0.8575$ $\therefore$ C

Q10



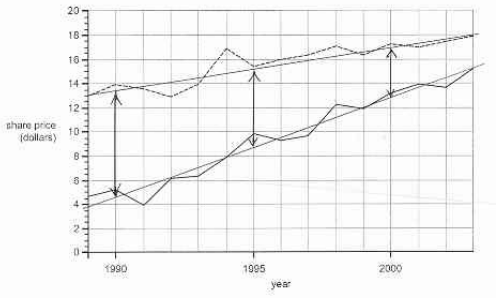
Slope $\approx \dfrac{57-7}{160-0} \approx 0.31$ $\therefore$ B

Q11



The redrawn line has the same slope because (x_L, y_L) and (x_U, y_U) remain the same. $\therefore$ E

Q12



Difference has a decreasing trend. $\therefore$ A

Q13 $\dfrac{actual}{index} = deseasonalised$,
$actual = index \times deseasonalised = 1.28 \times 28098$
$= 35965$ $\therefore$ D

SECTION B

Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
C	A	B	D	D	E	D	E	B

Q1 Harriet : Joshua : Ali : Total
5 : 3 : 2 : 10
6 : x

$\frac{x}{6}=\frac{10}{2}$, $x=30$ $\therefore$ C

Q2 Geometric sequence. $r=\frac{4.9}{7}=0.7$,

$t_4=t_3\times r=4.9\times 0.7=3.43$ $\therefore$ A

Q3 Infinite geometric series, $S_\infty=\frac{a}{1-r}$.

$a=-5.4$, $r=\frac{1.8}{-5.4}=-\frac{1}{3}$,

$S_\infty=\frac{-5.4}{1-\left(-\frac{1}{3}\right)}=-4.05$ $\therefore$ B

Q4 Start of 2003, number of goats = 600
Start of 2004, number of goats = 600 × 1.08
Start of 2005, number = 600 × 1.08 × 1.08 = 700
$\therefore$ D

Q5 Arithmetic sequence, $t_n=a+(n-1)d$.

$a=5$, $d=2$, $t_{12}=5+(12-1)\times 2=27$ $\therefore$ D

Q6 Arithmetic series, $S_n=\frac{n}{2}(2a+(n-1)d)$.

For one day each week,

$S_{20}=\frac{20}{2}(2\times 5+(20-1)\times 2)=480$.

For seven days each week, total number of laps
$= 7\times 480 = 3360$ $\therefore$ E

Q7 $w_3=4w_2+2=4\times 10+2=42$

$w_4=4w_3+2=4\times 42+2=170$ $\therefore$ D

Q8 Arithmetic sequence, $d=t_{n+1}-t_n=-5$, $a=15$,

$t_n=a+(n-1)d=15+(n-1)(-5)=20-5n$ $\therefore$ E

Q9 Amount of detergent $=\frac{20}{100}\times 200=40$ mL

Add V mL of water to the 200 mL of 20% detergent solution to make an 8% detergent solution,

$\therefore \frac{40}{200+V}\times 100\%=8\%$, $\frac{4000}{200+V}=8$

$4000=8(200+V)$, $V=300$ $\therefore$ B

Module 2: Geometry and trigonometry

1	2	3	4	5	6	7	8	9
E	B	C	E	D	C	A	D	A

Q1 $\sin\angle BAC=\frac{16}{25}$, $\angle BAC=40^o$ $\therefore$ E

Q2 $\angle YXZ=180-115-27=38^o$, $\therefore$ XY is north $90-38=52^o$ east. $\therefore$ B

Q3 The sine rule, $\frac{XZ}{\sin 115^o}=\frac{3.2}{\sin 27^o}$,

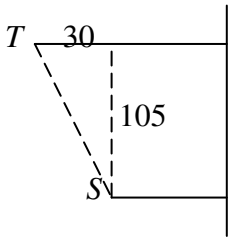
$XZ=6.4$ $\therefore$ C

Q4 Perimeter of the first Δ = 20 + 48 + 52 = 120

Perimeter of the second $\Delta=\frac{65}{52}\times 120=150$ $\therefore$ E

Q5 Actual area $=6^2\times 720=25920$ $\therefore$ D

Q6 $ST=\sqrt{30^2+105^2}$
$=109$
$\therefore$ C



Q7 Average slope $=\frac{rise}{run}=\frac{450-300}{200}=0.75$ $\therefore$ A

Q8 $\angle IFH=35-20=15^o$. The sine rule:

$\frac{FH}{\sin 20^o}=\frac{4}{\sin 15^o}$, $FH=\frac{4\sin 20^o}{\sin 15^o}$ $\therefore$ D

Q9 $AD=\sqrt{50^2+100^2}=111.8$, $\tan\angle ADB=\frac{20}{111.8}$,

$\angle ADB=\tan^{-1}\left(\frac{20}{111.8}\right)=10.1^o$ $\therefore$ A

Module 3: Graphs and relations

1	2	3	4	5	6	7	8	9
D	B	E	C	D	E	C	B	A

Q1 $1.00 + 0.50 = 1.50$ $\therefore$ D

Q2 $(2,1)$ is on the line $y = 3x + c$, $\therefore 1 = 3\times 2 + c$
$\therefore c = -5$ $\therefore$ B

Q3 Horizontal line $y = -8$ and vertical line $x = 12$ intersect at $(12,-8)$ $\therefore$ E

Q4 0 to 5, speed $= \dfrac{350}{5} = 70$; 5 to 9, speed = 0;
9 to 12, speed $= \dfrac{250}{3} = 83.3$; 12 to 14, speed = 0;
14 to 16, speed $= \dfrac{100}{2} = 50$. $\therefore$ C

Q5 $350 + 0 + 250 = 600$ $\therefore$ D

Q6 Hourly rate $= a = \dfrac{450-320}{6-4} = 65$
$\therefore C = 65x + b$, $\therefore 320 = 65\times 4 + b$, $b = 60$, hence
$C = 65x + 60$. For $x = 1$, $C = 125$ $\therefore$ E

Q7 Let the objective function be $P = y - 2x + 20$.
$(2,9)$, $P = 9 - 2(2) + 20 = 25$
$(4,11)$, $P = 11 - 2(4) + 20 = 23$
$(6,10)$, $P = 10 - 2(6) + 20 = 18$
$(6,1)$, $P = 1 - 2(6) + 20 = 9$ $\therefore$ C

Q8 The line with a negative slope is $y = -\dfrac{3}{4}x + 3$,
$\therefore$ the region below it is $y < -\dfrac{3}{4}x + 3$,
i.e. $3x + 4y < 12$. The line with a positive slope is $y = x - 1$, the region below it is $y < x - 1$, $1 < x - y$,
i.e. $x - y > 1$ $\therefore$ B

Q9 $y = kx^3$, $\therefore 1 = k(8)$, $k = \dfrac{1}{8}$, hence $y = \dfrac{1}{8}x^3$.
At $x = 2$, $y = 1$. $\therefore$ A

Module 4: Business-related mathematics

1	2	3	4	5	6	7	8	9
B	E	E	D	A	B	A	A	D

Q1 Simple interest $= 37000 \times \dfrac{4}{100} \times 2 = 2960$ $\therefore$ B

Q2 $R = 1 + \dfrac{3.5}{100} = 1.035$,
$A = PR^n = 150000(1.035)^6 = 184388.30$ $\therefore$ E

Q3 Charge $= 36 \times \left(1 + \dfrac{15}{100}\right) \times 2 = 82.80$ $\therefore$ E

Q4 Total cost $= 500 + 115 \times 24 = 3260$ $\therefore$ D

Q5 Interest $= 3260 - 2700 = 560$,
Interest rate $r = \dfrac{100I}{PT} = \dfrac{100 \times 560}{2700 \times 2} = 10.4$ $\therefore$ A

Q6 Monthly, $R = 1 + \dfrac{7.4}{100 \times 12} = 1.006167$. At the end of the term, $A = 0$, $\therefore 0 = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$,
$\therefore Q = \dfrac{PR^n(R-1)}{R^n - 1} = \dfrac{250000(1.006167)^{240}(0.006167)}{1.006167^{240} - 1}$,
$Q \approx 1999$ $\therefore$ B

Q7 $R = 1 + \dfrac{3.6}{100} = 1.036$.
After two years, $A = PR^2 = 12000(1.036)^2 = 12879.55$
During the third year, interest $= 12879.55 \times \dfrac{3.6}{100} \times 1$
$= 463.66$ $\therefore$ A

Q8 Due to depreciation the book value decreases and the annual depreciated amount decreases accordingly when the reducing balance method is used. $\therefore$ A

Q9 Monthly interest rate $\frac{7.5}{12} = 0.625$

Total number of repayment $= 12 \times 10 = 120$

Fully repaid over 10 years $\therefore$ no money will be owed after 10 years.

$$R = 1 + \frac{0.625}{100} = 1.00625$$

Monthly repayment $Q = \frac{PR^n(R-1)}{R^n - 1}$

$$= \frac{130000(1.00625)^{120}(0.00625)}{1.00625^{120} - 1} = 1543.10$$

Amount owing after 5 years $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$

$$= 130000(1.00625)^{60} - \frac{1543.12(1.00625^{60} - 1)}{1.00625 - 1}$$

$$= 77010.25$$

A monthly repayment of 1500 (which is less than 1543.10) will increase the length of the loan. $\therefore$ D

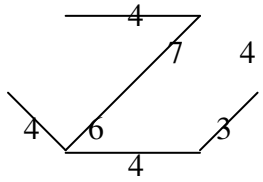
Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
B	E	C	A	B	A	E	D	D

Q1 A subgraph is a subset of the vertices and edges in a graph. $\therefore$ B

Q2 For any two there exists a one-one correspondence between their vertex sets that preserves adjacency. $\therefore$ E

Q3



$4 + 7 + 6 + 4 + 4 + 3 = 28$

$\therefore$ C

Q4 Without passing through the other towns, one route from *F* to *F*; one route from *F* to *I*; two routes from *F* to *G*; two routes from *F* to *H*; two routes from *G* to *H*; one route from *I* to *H*. $\therefore$ A

Q5 B

Q6 A critical path (path of maximum length from start to finish) determines the minimum time required to complete the entire project. $\therefore$ A

Q7 Use the Hungarian algorithm.

$$\begin{matrix} \\ -10 \\ -7 \\ -6 \\ -8 \\ -8 \end{matrix} \begin{bmatrix} 12 & 15 & 99 & 10 & 14 \\ 10 & 9 & 10 & 7 & 12 \\ 99 & 10 & 11 & 6 & 12 \\ 8 & 8 & 12 & 9 & 99 \\ 8 & 99 & 9 & 8 & 11 \end{bmatrix} \rightarrow \begin{matrix} \\ -1 \\ -3 \end{matrix} \begin{bmatrix} 2 & 5 & 89 & 0 & 4 \\ 3 & 2 & 3 & 0 & 5 \\ 93 & 4 & 7 & 0 & 6 \\ 0 & 0 & 4 & 1 & 91 \\ 0 & 91 & 1 & 0 & 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2 & 5 & 88 & 0 & 1 \\ 3 & 2 & 2 & 0 & 2 \\ 93 & 4 & 6 & 0 & 3 \\ 0 & 0 & 3 & 1 & 88 \\ 0 & 91 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 87 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 \\ 92 & 3 & 5 & 0 & 2 \\ 0 & 0 & 3 & 2 & 88 \\ 0 & 91 & 0 & 1 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 3 & 86 & 0 & (0) \\ 1 & (0) & 0 & 0 & 1 \\ 91 & 2 & 4 & (0) & 2 \\ (0) & 0 & 3 & 3 & 89 \\ 0 & 91 & (0) & 1 & 1 \end{bmatrix} \quad \therefore \text{E}$$

Q8 Critical path is: *Start-B-F-J-L-finish*.

Length $= 10 + 6 + 7 + 3 = 26$ $\therefore$ D

Q9 The latest start time for *E* is 2 hours before the earliest start time for *F*, i.e. 33, because time taken for *E* is 2 hours.

The earliest start time for *E* is 22, $\therefore$ time taken for *C* could be increased by 33 – 22 = 11 hours. $\therefore$ D

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

