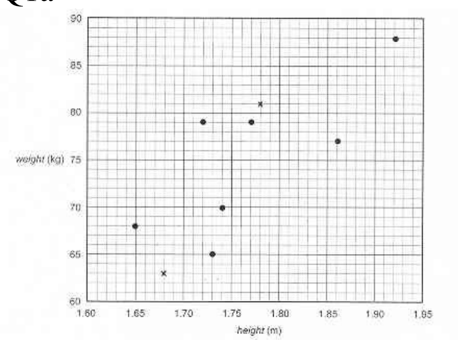
2004 VCAA Further Math Exam 2 Solutions

Core

Q1a



Q1b Graphics calculator.

$$weight = -60.8 + 76.8 \times height$$

Q1c weight; heights.

Q2a $BMI = \dfrac{66}{1.69^2} = 23.1$

Q2b $Range = BMI_{\max} - BMI_{\min} = 31.4 - 20.6 = 10.8$

Q2c

6	3
11	18

Q2d % of males overweight $= \dfrac{6}{17} \times 100\% = 35.3\%$

% of females overweight $= \dfrac{3}{21} \times 100\% = 14.3\%$

$\therefore$ yes.

Q2ei Approximately the same mean and inter-quartile range

Q2eii Mean BMI for males $= 23.9$

Q2eiii The mean is the sum of the data divided by the number of data. It takes into account every individual value of the data set. Median is the middle value of the sample and is not affected by the variations of other values in the data set as long as the median remains the median,

e.g. 1, 1, 2, 5, 6 median = 2, mean = 3
2, 2, 2, 7, 7 median = 2, mean = 4

$\therefore$ the mean gives a better indication.

Module 1: Number patterns and applications

Q1a $r = \dfrac{2200}{2000} = 1.1$

Q1b Annual % increase

$= \dfrac{2200 - 2000}{2000} \times 100\% = 10\%$

Q1c Number produced in year 5:

$t_5 = 2000 \times 1.1^4 = 2928$

Q1d Total in the first ten years:

$S_{10} = \dfrac{2000\left(1.1^{10} - 1\right)}{1.1 - 1} = 31875$

Q1e $P_1 = 2000$, $P_{n+1} = bP_n + c$

$P_2 = bP_1 + c = b(2000) + c = 2200$ (1)

$P_3 = bP_2 + c = b(2200) + c = 2420$ (2)

(2) – (1), $200b = 220$, $\therefore$ $b = 1.1$ (3)

Substitute (3) in (1), $2000 \times 1.1 + c = 2200$, $\therefore c = 0$

Q2a Cost $= 3500 + 3 \times 80 = \$3740$

Q2b $3500 + 80n \le 4400$, $80n \le 900$, $n \le 11.25$,

$\therefore$ greatest n = 11.

Q2c Number of outlets $= \dfrac{35}{20} \times 12 = 21$

Cost $= 3500 + (21 - 5) \times 80 = \$4780$

Q3a $S_n = 1.2S_{n-1} - 200$, $\therefore S_{n-1} = \dfrac{S_n + 200}{1.2}$,

$S_3 = 2224$, $S_2 = \dfrac{S_3 + 200}{1.2} = \dfrac{2224 + 200}{1.2} = 2020$,

$S_1 = \dfrac{S_2 + 200}{1.2} = \dfrac{2020 + 200}{1.2} = 1850$

Q3b

$\dfrac{S_1 + S_2 + S_3}{P_1 + P_2 + P_3} \times 100\% = \dfrac{1850 + 2020 + 2224}{2000 + 2200 + 2420} \times 100\%$
$= 92.1\%$

Module 2: Geometry and trigonometry

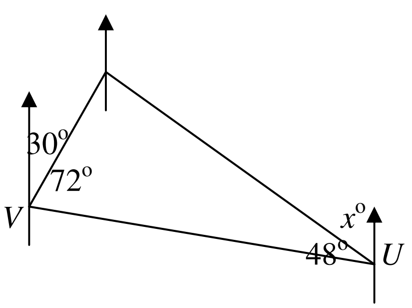
Q1a $\tan \angle ABC = \dfrac{10}{3.6}$, $\angle ABC = 70^o$

Q1b Pythagoras' theorem:
$\overline{BC} = \sqrt{10^2 + 3.6^2} = 10.6$ metres

Q1c The cosine rule:
$\overline{EF} = \sqrt{8.3^2 + 2.7^2 - 2(8.3)(2.7)\cos 130^o} = 10.2$ metres

Q1d
Area $= \dfrac{1}{2} ab \sin\theta^o = \dfrac{1}{2}(8.3)(2.7)\sin 130^o = 8.6\,\text{m}^2$

Q2a



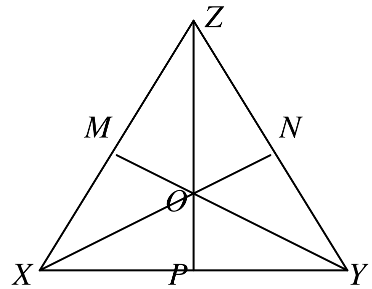
$x = 180 - 30 - 72 - 48 = 30$
$\therefore$ Bearing of V from U is 330° T.

Q2b The sine rule: $\dfrac{\overline{TU}}{\sin 72^o} = \dfrac{5.4}{\sin 48^o}$, $\overline{TU} = 6.9\,\text{km}$

Q2c $\angle VTU = 180 - 72 - 48 = 60$
The sine rule: $\dfrac{\overline{VU}}{\sin 60^o} = \dfrac{5.4}{\sin 48^o}$ $\therefore \overline{VU} = 6.3\,\text{km}$
$\therefore$ shortest distance $= 5.4 + 6.9 + 6.3 = 18.6\,\text{km}$

Q3a $\angle XYZ = \dfrac{180^o}{3} = 60^o$

Q3b Let P be the mid point of XY, ZOP is a straight line.



$\overline{XP} = 0.5$, $\overline{ZP} = \sqrt{1^2 - 0.5^2} = 0.8660$,
$\overline{OP} = \dfrac{1}{3} \times \overline{ZP} = \dfrac{1}{3} \times 0.8860 = 0.289\,\text{m}$

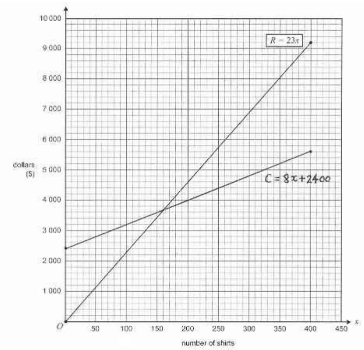
Q3c Area of $\Delta XYZ = \dfrac{1}{2} \times 1 \times 0.8660 = 0.433$
Area of circle $= \pi r^2$,
$\therefore$ Area of shaded region $= 0.433 - \pi r^2$
The ratio $= 2:1$
$\therefore (0.433 - \pi r^2):(\pi r^2) = 2:1$, $\dfrac{0.433 - \pi r^2}{\pi r^2} = \dfrac{2}{1}$,
$0.433 - \pi r^2 = 2\pi r^2$, $3\pi r^2 = 0.433$, $r = 0.214\,\text{m}$

Module 3: Graphs and relations

Q1a When $x = 400$, $C = 8 \times 400 + 2400 = \$5600$.

Q1b $8x + 2400 \le 3000$, $8x \le 600$, $x \le 75$
$\therefore$ max number of shirts = 75.

Q1c



Q1d Read from graph, $x = 160$

Q1e $P = R - C$, $P = 23x - (8x + 2400)$,
$P = 15x - 2400$

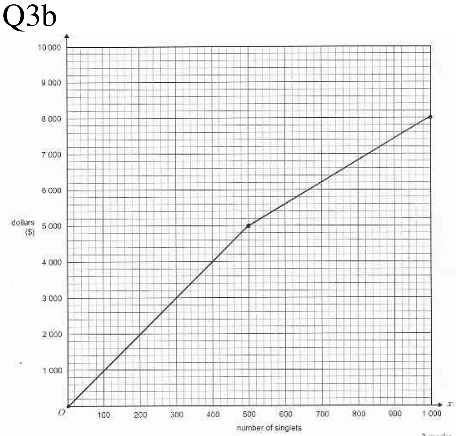
Q1f When $x = 345$, $P = 15 \times 345 - 2400 = \$2775$

Q2 $R = C + P = 4800 + 3000 = 7800$

Selling price for each $= \dfrac{7800}{250} = \$31.20$

Q3a For $x > 500$, $R_S = 6x + 2000$.
When $x = 620$, $R_S = 6 \times 620 + 2000 = \$5720$

Q3b



Q3c

$$P = R_S - C_S = \begin{cases} 10x - (4x + 1500) = 6x - 1500 \\ 6x + 2000 - (4x + 1500) = 2x + 500 \end{cases}$$

The first equation is for $x \le 500$, and the second equation for $x > 500$.

To obtain $P = 2000$,
From first equation, $2000 = 6x - 1500$, $x = 583$ and $\therefore$ the first equation is not to be used.
From second equation, $2000 = 2x + 500$, $x = 750$ which is > 500. $\therefore$ number of singlets is 750.

Module 4: Business-related mathematics

Q1ai $120 \times 6 = \$720$

Q1aii $720 - 650 = \$70$

Q1bi $r = \dfrac{100I}{Pt} = \dfrac{100 \times 70}{650 \times 0.5} = 21.5$

Q1bii Because the same interest rate is applied on the initial amount throughout the term of the loan even though the principal is reduced for each monthly repayment.

Q1c Let P be the price before GST,
$\therefore \left(1 + \dfrac{10}{100}\right)P = 650$, $P = \$590.91$.

Q1d Flat rate method:

Total depreciation $= 650 \times \dfrac{12}{100} \times 5 = \$390$

Reducing balance method:

Total depreciation $= 650 - 650 \times \left(1 - \dfrac{15}{100}\right)^5 = \$288.41$

$\therefore$ Flat rate method gives greater total depreciation over five years.

Q2a $A = 0$, $n = 4 \times 12 = 48$, $P = 12000$

Q2b $R = 1 + \dfrac{7.5}{100 \times 12} = 1.00625$

For $A = 0$, $Q = \dfrac{PR^n (R - 1)}{R^n - 1}$

$= \dfrac{12000(1.00625)^{48}(1.00625 - 1)}{1.00625^{48} - 1} = \$290.15$

Q2c $I = 290.15 \times 48 - 12000 = \$1927.05$

Q2d $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$

$= 12000(1.00625)^6 - \dfrac{290.15(1.00625^6 - 1)}{1.00625 - 1} = 10688.76$

Anna paid off the loan by $12000 - 10688.76 \approx \$1311$

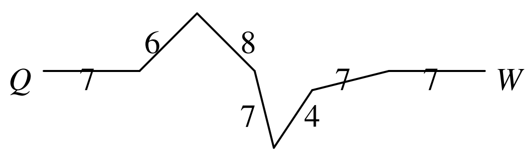
Q2ei $n = 42$, $P = \$10688.76$

Q2eii $R = 1 + \frac{8.0}{100 \times 12} = 1.006667$

$$Q = \frac{10688.76(1.006667)^{42}(1.006667 - 1)}{1.006667^{42} - 1} \approx \$293$$

Module 5: Networks and decision mathematics

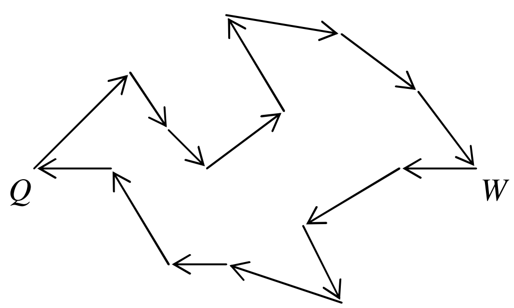
Q1ai



Q1aii 7 + 6 + 8 + 7 + 4 + 7 + 7 = 46 km

Q1bi A Hamilton circuit

Q1bii



Q2a

A			1
D	0		
F			10
K	12		

Q2b Start-B-C-E-G-J-K-finish,

Q2c 5 + 2 + 6 + 2 = 15 hours

Q2di Activities B and F because they are part of the critical path for the modified project. A remains the same because the earliest start time for F is 6. E also remains the same because the earliest start time for I is 10.

Q2dii 1 + 2 = 3 hours

Q2diii $1 \times 100 + 2 \times 50 = \$200$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors