Mathematical Methods 3,4 Summary sheets

Distance between two points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Mid-point $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

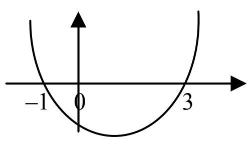
Parallel lines, $m_1 = m_2$

Perpendicular lines,

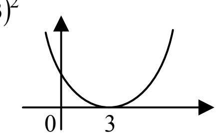
$m_1 m_2 = -1$ or $m_2 = -\frac{1}{m_1}$

Graphs of polynomial functions in factorised form:

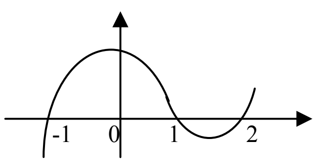
Quadratics e.g. $y = (x+1)(x-3)$



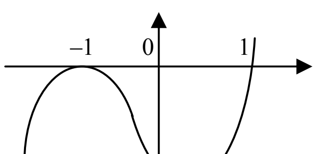
e.g. $y = (x-3)^2$



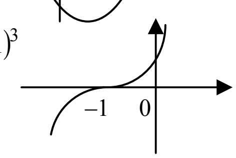
Cubics e.g. $y = 3(x+1)(x-1)(x-2)$



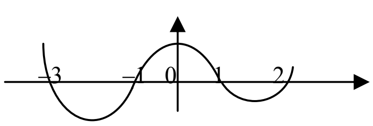
e.g. $y = (x+1)^2(x-1)$



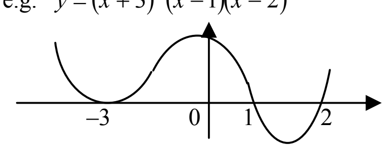
e.g. $y = (x+1)^3$



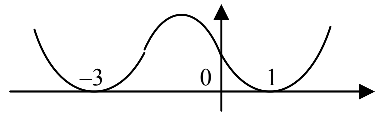
Quartics e.g. $y = (x+3)(x+1)(x-1)(x-2)$



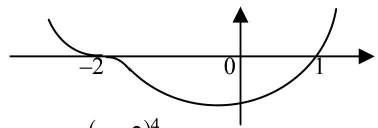
e.g. $y = (x+3)^2(x-1)(x-2)$



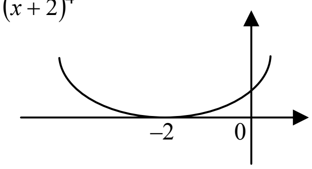
e.g. $y = (x+3)^2(x-1)^2$



e.g. $y = (x+2)^3(x-1)$

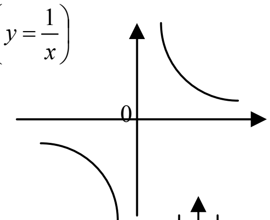


e.g. $y = (x+2)^4$

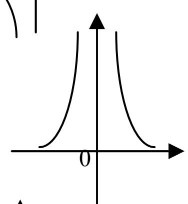


Examples of power functions:

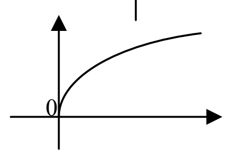
$y = x^{-1}$ $\left(y = \frac{1}{x} \right)$



$y = x^{-2}$ $\left(y = \frac{1}{x^2} \right)$

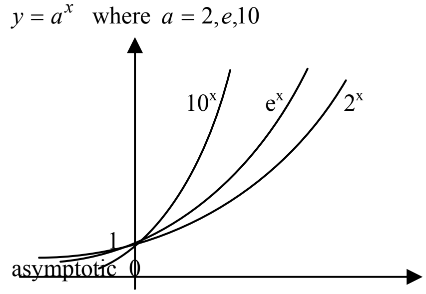


$y = x^{\frac{1}{2}}$ $\left(y = \sqrt{x} \right)$



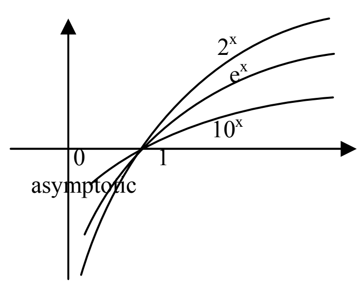
Exponential functions:

$y = a^x$ where $a = 2, e, 10$



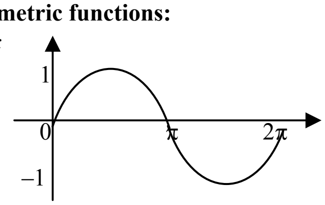
Logarithmic functions:

$y = \log_a x$ where $a = 2, e, 10$

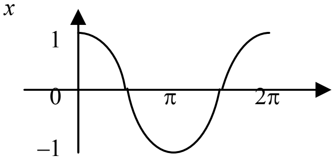


Trigonometric functions:

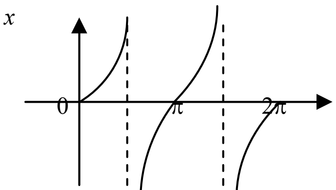
$y = \sin x$



$y = \cos x$

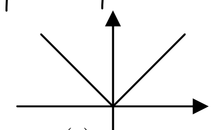


$y = \tan x$



Modulus functions

$y = |x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$



Transformations of $y = f(x)$

(1) Vertical dilation (dilation away from the x-axis, dilation parallel to the y-axis) by factor *k*. $y = kf(x)$

(2) Horizontal dilation (dilation away from the y-axis, dilation parallel to the x-axis) by factor $\frac{1}{n}$. $y = f(nx)$

(3) Reflection in the x-axis. $y = -f(x)$

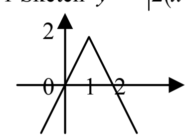
(4) Reflection in the y-axis. $y = f(-x)$

(5) Vertical translation (translation parallel to the y-axis) by *c* units.
$y = f(x) \pm c$, + up, – down.

(6) Horizontal translation (translation parallel to the x-axis) by *b* units.
$y = f(x \pm b)$, + left, – right.

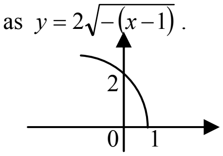
*Always carry out translations last in sketching graphs.

Example 1 Sketch $y = -|2(x-1)| + 2$



Example 2 Sketch $y = 2\sqrt{1-x}$.

Rewrite as $y = 2\sqrt{-(x-1)}$.



Relations and functions:

A relation is a set of ordered pairs (points). If no two ordered pairs have the same first element, then the relation is a function.

*Use the vertical line test to determine whether a relation is a function.

*Use the horizontal line test to determine whether a function is one-to-one or many-to-one.

*The inverse of a relation is given by its reflection in the line $y = x$.

*The inverse of a one-to-one function is a function and is denoted by f^{-1}. The inverse of a many-to-one function is **not** a function and therefore cannot be called inverse function, and f^{-1} cannot be used to denote the inverse.

Factorisation of polynomials:

(1) Check for common factors first.

(2) Difference of two squares,

e.g. $x^4-9=\left(x^2\right)^2-3^2=\left(x^2-3\right)\left(x^2+3\right)$
$=\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)\left(x^2+3\right)$

(3) Trinomials, by trial and error,

e.g. $2x^2-x-1=(2x+1)(x-1)$

(4) Difference of two cubes, e.g.
$x^3-y^3=(x-y)\left(x^2+xy+y^2\right)$

(5) Sum of two cubes, e.g. $8+a^3=$
$2^3+a^3=(2+a)\left(4-2a+a^2\right)$

(6) Grouping two and two,

e.g. $x^3+3x^2+3x+1=\left(x^3+1\right)+\left(3x^2+3x\right)$
$=(x+1)\left(x^2-x+1\right)+3x(x+1)$
$=(x+1)\left(x^2-x+1+3x\right)$
$=(x+1)\left(x^2+2x+1\right)=(x+1)^3$

(7) Grouping three and one,

e.g. x^2-2x-y^2+1
$=\left(x^2-2x+1\right)-y^2=(x-1)^2-y^2$
$=(x-1-y)(x-1+y)$

(8) Completing the square, e.g.

$$x^2+x-1=x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-1$$

$$=\left(x^2+x+\frac{1}{4}\right)-\frac{5}{4}=\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{5}}{2}\right)^2$$

$$=\left(x+\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)$$

(9) **Factor theorem**,

e.g. $P(x)=x^3-3x^2+3x-1$
$P(-1)=(-1)^3-3(-1)^2+3(-1)-1\neq 0$
$P(1)=1^3-3(1)^2+3(1)-1=0$
$\therefore (x-1)$ is a factor.

Long division:

$$\begin{array}{r} x^2-2x+1 \\ x-1\overline{)x^3-3x^2+3x-1} \\ -\left(x^3-x^2\right) \\ \hline -2x^2+3x \\ -\left(-2x^2+2x\right) \\ \hline x-1 \\ -(x-1) \\ \hline 0 \end{array}$$

$\therefore P(x)=(x-1)\left(x^2-2x+1\right)=(x-1)^3$

Remainder theorem:

e.g. when $P(x)=x^3-3x^2+3x-1$ is divided by $x+2$, the remainder is
$P(-2)=(-2)^3-3(-2)^2+3(-2)-1=-11$

When it is divided by $2x-3$, the remainder is $P\left(\frac{3}{2}\right)=\frac{1}{8}$.

Quadratic formula:

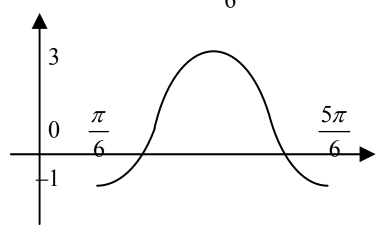
Solutions of $ax^2+bx+c=0$ are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Graphs of transformed trig. functions

e.g. $y=-2\cos\left(3x-\frac{\pi}{2}\right)+1$, rewrite equation as $y=-2\cos 3\left(x-\frac{\pi}{6}\right)+1$.

The graph is obtained by reflecting it in the x-axis, dilating it vertically so that its amplitude becomes 2, dilating it horizontally so that its period becomes $\frac{2\pi}{3}$, translating upwards by 1 and right by $\frac{\pi}{6}$.



Solving trig. equations

e.g. Solve $\sin 2x=\frac{\sqrt{3}}{2}$, $0\le x\le 2\pi$.

$\therefore 0\le 2x\le 4\pi$,

$2x=\frac{\pi}{3},\frac{2\pi}{3},\frac{\pi}{3}+2\pi,\frac{2\pi}{3}+2\pi$

$\therefore x=\frac{\pi}{6},\frac{\pi}{3},\frac{7\pi}{6},\frac{4\pi}{3}.$

e.g. $\sin\frac{x}{2}=\sqrt{3}\cos\frac{x}{2}$, $0\le x\le 2\pi$.

$0\le\frac{x}{2}\le\pi$, $\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\sqrt{3}$, $\tan\frac{x}{2}=\sqrt{3}$,

$\therefore \frac{x}{2}=\frac{\pi}{3}$, $\therefore x=\frac{2\pi}{3}$.

Exact values for trig. functions:

x^o	x	$\sin x$	$\cos x$	$\tan x$
0	0	0	1	0
30	π/6	1/2	√3/2	1/√3
45	π/4	1/√2	1/√2	1
60	π/3	√3/2	1/2	√3
90	π/2	1	0	undef
120	2π/3	√3/2	–1/2	–√3
135	3π/4	1/√2	–1/√2	–1
150	5π/6	1/2	–√3/2	–1/√3
180	π	0	–1	0
210	7π/6	–1/2	–√3/2	1/√3
225	5π/4	–1/√2	–1/√2	1
240	4π/3	–√3/2	–1/2	√3
270	3π/2	–1	0	undef
300	5π/3	–√3/2	1/2	–√3
315	7π/4	–1/√2	1/√2	–1
330	11π/6	–1/2	√3/2	–1/√3
360	2π	0	1	0

Index laws:

$a^m a^n=a^{m+n}, \frac{a^m}{a^n}=a^{m-n}, \left(a^m\right)^n=a^{mn}$

$(ab)^n=a^n b^n, \frac{1}{a^n}=a^{-n}, a^m=\frac{1}{a^{-m}}$

$a^0=1, a^{\frac{1}{2}}=\sqrt{a}, a^{\frac{1}{n}}=\sqrt[n]{a}$

Logarithm laws:

$\log a+\log b=\log ab, \log a-\log b=\log\frac{a}{b}$

$\log a^b=b\log a, \log\frac{1}{b}=-\log b, \log_a a=1$

$\log 1=0, \log 0=undef, \log(neg)=undef$

Change of base:

$\log_a x=\frac{\log_b x}{\log_b a}$,

e.g. $\log_2 7=\frac{\log_e 7}{\log_e 2}=2.8$.

Exponential equations:

e.g. $2e^{3x}=5$, $e^{3x}=2.5$, $3x=\log_e 2.5$,
$x=\frac{1}{3}\log_e 2.5$

e.g. $2e^{2x}-3e^x-2=0$,
$2\left(e^x\right)^2-3\left(e^x\right)-2=0$,
$\left(2e^x+1\right)\left(e^x-2\right)=0$, since $2e^x+1\neq 0$,
$\therefore e^x-2=0$, $e^x=2$, $x=\log_e 2$.

Equations involving log:

e.g. $\log_e(1-2x)+1=0$, $\log_e(1-2x)=-1$,
$1-2x=e^{-1}$, $2x=1-e^{-1}$, $x=\frac{1}{2}\left(1-\frac{1}{e}\right)$.

e.g. $\log_{10}(x-1)=1-\log_{10}(2x-1)$
$\log_{10}(x-1)+\log_{10}(2x-1)=1$
$\log_{10}(x-1)(2x-1)=1$, $(x-1)(2x-1)=10$,
$2x^2-3x-9=0$, $(2x+3)(x-3)=0$,
$x=-\frac{3}{2}$, 3. 3 is the only solution because $x=-\frac{3}{2}$ makes the log equation undefined.

Equation of inverse:

Interchange *x* and *y* in the equation to obtain the equation of the inverse. If possible express *y* in terms of *x*.

e.g. $y=2(x-1)^2+1$, $x=2(y-1)^2+1$,
$2(y-1)^2=x-1$, $(y-1)^2=\frac{x-1}{2}$,
$y=\pm\sqrt{\frac{x-1}{2}}+1$.

e.g. $y=-\frac{2}{x-1}+4$, $x=-\frac{2}{y-1}+4$,
$x-4=-\frac{2}{y-1}$, $y-1=-\frac{2}{x-4}$,
$y=-\frac{2}{x-4}+1$.

e.g. $y=-2e^{x-1}+1$, $x=-2e^{y-1}+1$,
$2e^{y-1}=1-x$, $e^{y-1}=\frac{1-x}{2}$,
$y-1=\log_e\left(\frac{1-x}{2}\right)$, $y=\log_e\left(\frac{1-x}{2}\right)+1$.

e.g. $y=-\log_e(1-2x)-1$,
$x=-\log_e(1-2y)-1$,
$\log_e(1-2y)=-(x+1)$, $1-2y=e^{-(x+1)}$
$2y=1-e^{-(x+1)}$, $y=\frac{1}{2}\left(1-e^{-(x+1)}\right)$.

The binomial theorem:
e.g. Expand $(2x-1)^4$
$={}^4C_0(2x)^4(-1)^0+{}^4C_1(2x)^3(-1)^1$
$+{}^4C_2(2x)^2(-1)^2+{}^4C_3(2x)^1(-1)^3$
$+{}^4C_4(2x)^0(-1)^4=......$
e.g. Find the coefficient of x^2 in the expansion of $(2x-3)^5$.
The required term is ${}^5C_3(2x)^2(-3)^3$
$=10(4x^2)(-27)=-1080x^2$.
$\therefore$ the coefficient of x^2 is -1080.

Differentiation rules:

$y=f(x)$	$\frac{dy}{dx}=f'(x)$
ax^n	anx^{n-1}
$a(x+c)^n$	$an(x+c)^{n-1}$
$a(bx+c)^n$	$abn(bx+c)^{n-1}$
$a\sin x$	$a\cos x$
$a\sin(x+c)$	$a\cos(x+c)$
$a\sin(bx+c)$	$ab\cos(bx+c)$
$a\cos x$	$-a\sin x$
$a\cos(x+c)$	$-a\sin(x+c)$
$a\cos(bx+c)$	$-ab\sin(bx+c)$
$a\tan x$	$a\sec^2 x$
$a\tan(x+c)$	$a\sec^2(x+c)$
$a\tan(bx+c)$	$ab\sec^2(bx+c)$
ae^x	ae^x
ae^{x+c}	ae^{x+c}
ae^{bx+c}	abe^{bx+c}
$a\log_e x$	$\frac{a}{x}$
$a\log_e bx$	$\frac{a}{x}$
$a\log_e(x+c)$	$\frac{a}{x+c}$
$a\log_e b(x+c)$	$\frac{a}{x+c}$
$a\log_e(bx+c)$	$\frac{ab}{bx+c}$

Differentiation rules:
The product rule: For the multiplication of two functions, $y=u(x)v(x)$, e.g.
$y=x^2\sin 2x$, let $u=x^2$, $v=\sin 2x$,
$\frac{dy}{dx}=v\frac{du}{dx}+u\frac{dv}{dx}$
$=(\sin 2x)(2x)+(x^2)(2\cos 2x)$
$=2x(\sin 2x+x\cos 2x)$
The quotient rule: For the division of functions, $y=\frac{u(x)}{v(x)}$, e.g. $y=\frac{\log_e x}{x}$,

$$\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$=\frac{(x)\left(\frac{1}{x}\right)-(\log_e x)(1)}{x^2}=\frac{1-\log_e x}{x^2}.$$

The chain rule: For composite functions, $y=f(u(x))$, e.g. $y=e^{\cos x}$.
Let $u=\cos x$, $y=e^u$, $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
$=(e^u)(-\sin x)=-e^{\cos x}\sin x$.

Finding stationary points: Let $\frac{dy}{dx}=0$ and solve for x and then y, the coordinates of the stationary point.
Nature of stationary point at $x=a$:

	Local max.	Local min.	Inflection point
$x<a$	$\frac{dy}{dx}>0$	$\frac{dy}{dx}<0$	$\frac{dy}{dx}>0$, (<0)
$x=a$	$\frac{dy}{dx}=0$	$\frac{dy}{dx}=0$	$\frac{dy}{dx}=0$
$x>a$	$\frac{dy}{dx}<0$	$\frac{dy}{dx}>0$	$\frac{dy}{dx}>0$, (<0)

Equation of tangent and normal at $x=a$:
1) Find the y coordinate if it is not given.
2) Gradient of tangent $m_T=\frac{dy}{dx}$ at $x=a$.
3) Use $y-y_1=m_T(x-x_1)$ to find equation of tangent.
4) Find gradient of normal $m_N=-\frac{1}{m_T}$.
5) Use $y-y_1=m_N(x-x_1)$ to find equation of the normal.

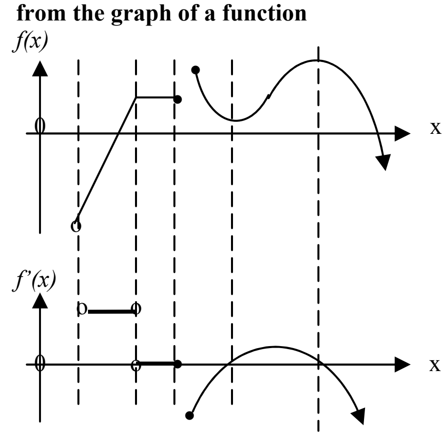
Linear approximation:
To find the approx. value of a function, use $f(a+h)\approx f(a)+hf'(a)$, e.g. find the approx. value of $\sqrt{25.1}$. Let $f(x)=\sqrt{x}$, then $f'(x)=\frac{1}{2\sqrt{x}}$. Let $a=25$ and $h=0.1$, then $f(a+h)=\sqrt{25.1}$, $f(a)=\sqrt{25}=5$, $f'(a)=\frac{1}{2\sqrt{25}}=0.1$.
$\therefore\sqrt{25.1}\approx 5+0.1\times 0.1=5.01$

The approx. change in a function is $=f(a+h)-f(a)=hf'(a)$,
e.g. find the approx. change in $\cos x$ when x changes from $\frac{\pi}{2}$ to 1.6. Let $f(x)=\cos x$, then $f'(x)=-\sin x$. Let $a=\frac{\pi}{2}$, then $f'(a)=-\sin\frac{\pi}{2}=-1$ and $h=1.6-\frac{\pi}{2}=0.03$
Change in $\cos x=hf'(a)=0.03\times{}^{-}1=-0.03$

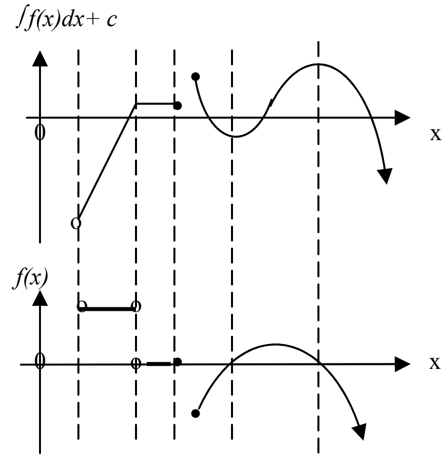
Rate of change: $\frac{dy}{dx}$ is the rate of change of y with respect to x. $v=\frac{dx}{dt}$, velocity is the rate of change of position x with respect to time t. $a=\frac{dv}{dt}$, acceleration a is the rate of change of velocity v with respect to t.

Average rate of change: Given $y=f(x)$, when $x=a$, $y=f(a)$, when $x=b$, $y=f(b)$, the average rate of change of y with respect to x $=\frac{\Delta y}{\Delta x}=\frac{f(b)-f(a)}{b-a}$.

Deducing the graph of gradient function from the graph of a function



Deducing the graph of function from the graph of anti-derivative function



Anti-differentiation (indefinite integrals):

$f(x)$	$\int f(x)dx$
ax^n for $n \neq -1$	$\frac{a}{n+1}x^{n+1}$
$a(x+c)^n$, $n \neq -1$	$\frac{a}{n+1}(x+c)^{n+1}$
$a(bx+c)^n$, $n \neq -1$	$\frac{a}{(n+1)b}(bx+c)^{n+1}$
$\frac{a}{x}$	$a\log_e x$, $x>0$ $a\log_e(-x)$, $x<0$
$\frac{a}{x+c}$	$a\log_e(x+c)$
$\frac{a}{bx+c}$	$\frac{a}{b}\log_e(bx+c)$
ae^x	ae^x
ae^{x+c}	ae^{x+c}
ae^{bx+c}	$\frac{a}{b}e^{bx+c}$
$a\sin x$	$-a\cos x$
$a\sin(x+c)$	$-a\cos(x+c)$
$a\sin(bx+c)$	$-\frac{a}{b}\cos(bx+c)$
$a\cos x$	$a\sin x$
$a\cos(x+c)$	$a\sin(x+c)$
$a\cos(bx+c)$	$\frac{a}{b}\sin(bx+c)$

Definite integrals:

e.g. $\int_0^{\frac{\pi}{2}}\cos\left(x-\frac{\pi}{3}\right)dx = \left[\sin\left(x-\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{2}}$

$= \sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)-\sin\left(0-\frac{\pi}{3}\right)$

$= \sin\frac{\pi}{6}-\sin\left(-\frac{\pi}{3}\right) = \frac{1+\sqrt{3}}{2}$.

Properties of definite integrals:

1) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

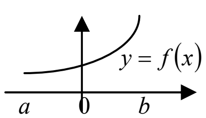
2) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$,

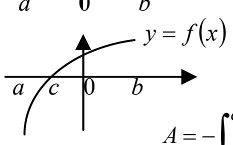
where $a<c<b$. 4) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

4) $\int_a^b f(x)dx = -\int_b^a f(x)dx$, 5) $\int_a^a f(x)dx = 0$.

Area 'under' curve:

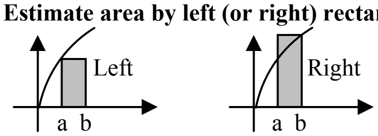


$A = \int_a^b f(x)dx$

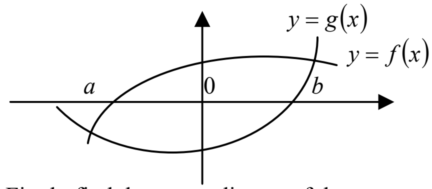


$A = -\int_a^c f(x)dx + \int_c^b f(x)dx$

Estimate area by left (or right) rectangles



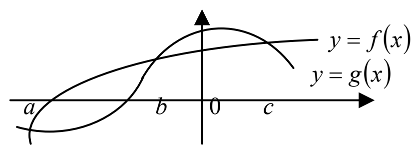
Area between two curves:



Firstly find the x-coordinates of the intersecting points, *a*, *b*, then evaluate

$A = \int_a^b [f(x)-g(x)]dx$. Always the function above minus the function below.

For three intersecting points:



$A = \int_a^b [f(x)-g(x)]dx + \int_b^c [g(x)-f(x)]dx$

Discrete probability distributions:

In general, in the form of a table,

x	x_1	x_2	x_3	......
$\Pr(X=x)$	p_1	p_2	p_3	......

$p_1, p_2, p_3, ...$ have values from 0 to 1 and $p_1+p_2+p_3+...=1$.

$\mu = E(X) = x_1p_1 + x_2p_2 + x_3p_3 + ...$

$Var(X) = x_1^2p_1 + x_2^2p_2 + x_3^2p_3 + ... - \mu^2$

$\sigma = sd(X) = \sqrt{Var(X)}$

If random variable $Y = aX + b$,

$E(Y) = aE(X) + b$, $Var(Y) = a^2 \times Var(X)$

and $sd(Y) = a \times sd(X)$.

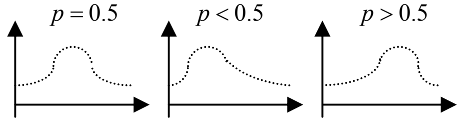
95% probability interval : $(\mu - 2\delta, \mu + 2\delta)$

Conditional prob: $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

Binomial distributions are examples of discrete prob. distributions. Sampling with replacement has a binomial distribution. Number of trials = *n*. In a single trial, prob. of success = *p*, prob. of failure = *q* = 1- *p*. The random variable *X* is the number of successes in the *n* trials. The binomial dist. is $\Pr(X=x) = {}^nC_x p^x q^{n-x}$, $x = 0,1,2,...$ with $\mu = np$ and $\sigma = \sqrt{npq} = \sqrt{np(1-p)}$.

*** Effects of increasing n on the graph of a binomial distribution.* (1) more points (2) lower probability for each *x* value (3) becoming symmetrical , bell shape.

*** Effects of changing p on the graph of a binomial distribution.* (1) bell shape when $p = 0.5$ (2) positively skewed if $p < 0.5$ (3) negatively skewed if $p > 0.5$



Graphics calculator :

$\Pr(X = a) = binompdf(n, p, a)$

$\Pr(X \le a) = binomcdf(n, p, a)$

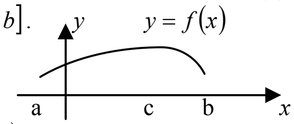
$\Pr(X < a) = binomcdf(n, p, a-1)$

$\Pr(X \ge a) = 1 - binomcdf(n, p, a-1)$

$\Pr(X > a) = 1 - binomcdf(n, p, a)$

$\Pr(a \le X \le b) = binomcdf(n, p, b) - binomcdf(n, p, a-1)$

Probability density functions $f(x)$ for $x \in [a, b]$.



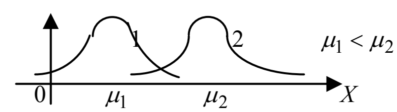
For $f(x)$ to be a probability density function, $f(x) > 0$ and

$\Pr(a < X < b) = \int_a^b f(x)dx = 1$.

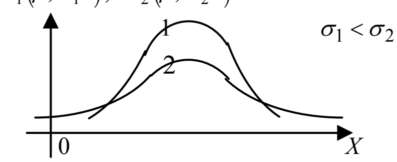
$\Pr(X < c) = \int_a^c f(x)dx$, $\Pr(X > c) = \int_c^b f(x)dx$

Normal distributions are continuous prob. distributions. The graph of a normal dist. has a bell shape and the area under the graph represents probability. Total area = 1.

$N_1(\mu_1, \sigma^2)$, $N_2(\mu_2, \sigma^2)$.

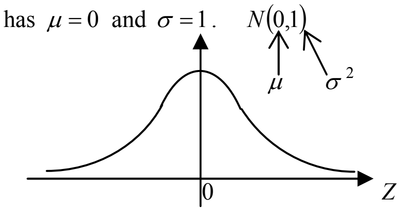


$N_1(\mu, \sigma_1^2)$, $N_2(\mu, \sigma_2^2)$.



The standard normal distribution:

has $\mu = 0$ and $\sigma = 1$. $N(0,1)$



Graphics calculator: Finding probability,

$\Pr(X < a) = normalcdf(-E99, a, \mu, \sigma)$

$\Pr(X > a) = normalcdf(a, E99, \mu, \sigma)$

$\Pr(a < X < b) = normalcdf(a, b, \mu, \sigma)$

Finding quantile, e.g. given $\Pr(X < x) = 0.7$

$x = invNorn(0.7, \mu, \sigma)$.

Given $\Pr(X > x) = 0.7$, then

$\Pr(X < x) = 1 - 0.7 = 0.3$ and

$x = invNorm(0.3, \mu, \sigma)$.

To find μ and/or σ, use $Z = \frac{X-\mu}{\sigma}$ to convert *X* to *Z* first, e.g. find μ given $\sigma = 2$ and $\Pr(X < 4) = 0.8$. $\Pr\left(Z < \frac{4-\mu}{2}\right) = 0.8$,

$\therefore \frac{4-\mu}{2} = invNorm(0.8) = 0.8416$,

$\therefore \mu = 2.3168$.