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Mathematical Methods 3/4 Trial Exam 1 Solutions 2007
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1a. $Z = \frac{45-56}{5} = -2.2$

1bi.
$\Pr(56 < X < 61) = \Pr(0 < Z < 1) = \frac{1}{2}\Pr(-1 < Z < 1) = \frac{1}{2} \times 0.68 = 0.34$

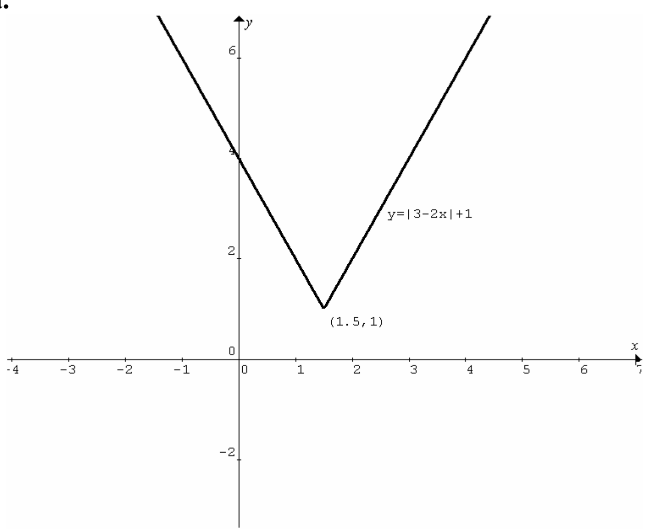
1bii.
$\Pr(X < 51 \cup X > 61)$
$= \Pr(Z < -1 \cup Z > 1) = 1 - \Pr(-1 < Z < 1) = 1 - 0.68 = 0.32$

2a. $\int_0^3 kx\,dx = 1$, $\left[\frac{kx^2}{2}\right]_0^3 = 1$, $\therefore \frac{9k}{2} = 1$, $k = \frac{2}{9}$

2b. $\Pr(X > 2) = \int_2^3 \frac{2}{9}x\,dx = \left[\frac{x^2}{9}\right]_2^3 = 1 - \frac{4}{9} = \frac{5}{9}$

3. Solve $x^3 - 3x^2 - 8x + 24 = 0$, $x \ge 0$.
$(x^3 - 3x^2) - (8x - 24) = 0$, $x^2(x-3) - 8(x-3) = 0$,
$(x^2 - 8)(x-3) = 0$, $(x - 2\sqrt{2})(x + 2\sqrt{2})(x-3) = 0$.
Since $x \ge 0$, $\therefore x = 2\sqrt{2}$ or 3

4a.



4b.
Translate 1 unit downwards.
Translate 1.5 units to the left.
Dilate horizontally by a factor of 2 (or dilate vertically by a factor of $\frac{1}{2}$.

5a. $f(x) = 10^{2\log_{10}(x) - \log_{10}(x+1) - 1} = 10^{\log_{10}(x^2) - \log_{10}(x+1) - \log_{10}10}$
$= 10^{\log_{10}\frac{x^2}{10(x+1)}} = \frac{x^2}{10(x+1)} = \frac{x^2}{10x+10}$ where $x > 0$

5b. Equation of the original function: $y = \frac{x^2}{10x+10}$ where $x > 0$

Equation of the inverse: $x = \frac{y^2}{10y+10}$ where $y > 0$

Transpose to make y the subject of the equation:
$y^2 - 10xy - 10x = 0$,
$y = \frac{10x + \sqrt{100x^2 + 40x}}{2}$ (quadratic formula: $a = 1$, $b = -10x$, $c = -10x$),
$\therefore y = 5x + \sqrt{25x^2 + 10x}$ (Note: Only '+' results in $y > 0$)
$\therefore f^{-1}(x) = 5x + \sqrt{25x^2 + 10x}$.

6. $\sin(3\theta) + \sqrt{3}\cos(3\theta) = 0$, $-\pi \le \theta \le \pi$
$\frac{\sin(3\theta) + \sqrt{3}\cos(3\theta)}{\cos(3\theta)} = 0$, $-3\pi \le 3\theta \le 3\pi$
$\frac{\sin(3\theta)}{\cos(3\theta)} + \sqrt{3} = 0$, $\tan(3\theta) = -\sqrt{3}$,
$\therefore 3\theta = -\frac{7\pi}{3}, -\frac{4\pi}{3}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$
$\therefore \theta = -\frac{7\pi}{9}, -\frac{4\pi}{9}, -\frac{\pi}{9}, \frac{2\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}$.

7a. $e^{2x} - e^x - 2 = 0$, $(e^x - 2)(e^x + 1) = 0$.
Since $e^x + 1 > 0$, $\therefore e^x - 2 = 0$, $e^x = 2$, $x = \log_e 2$.

7b. When $e^x = 2$, $x = \log_e 2$ and $y = 2^2 - 2 - 2 = 0$,
$\therefore (\log_e 2, 0)$.
Gradient function $= \frac{dy}{dx} = 2e^{2x} - e^x$.
When $e^x = 2$, gradient of tangent $= m_T = 2(2^2) - 2 = 6$.
$\therefore$ gradient of normal $= m_N = -\frac{1}{m_T} = -\frac{1}{6}$.
Equation of normal at $(\log_e 2, 0)$:
$y - y_1 = m_N(x - x_1)$,
$y = -\frac{1}{6}(x - \log_e 2)$,
$x + 6y = \log_e 2$.

8. Area $= -\int_p^{-1} \frac{2}{x} dx$, $-\int_p^{-1} \frac{2}{x} dx = 4$, where $p < 0$.

$-\left[2\log_e|x|\right]_p^{-1} = 4$, $-2\log_e|-1| + 2\log_e|p| = 4$,

$-2\log_e 1 + 2\log_e(-p) = 4$,

$2\log_e(-p) = 4$, $\log_e(-p) = 2$,

$-p = e^2$, $\therefore p = -e^2$.

9a. $y = f(x) = -3x^4 - 4x^3 + 6x^2 + 12x - 3$,

$f'(x) = -12x^3 - 12x^2 + 12x + 12$.

At stationary points, $f'(x) = 0$, $-12x^3 - 12x^2 + 12x + 12 = 0$,

$-12(x^3 + x^2 - x - 1) = 0$, $-12((x^3 + x^2) - (x+1)) = 0$,

$-12(x^2(x+1) - 1(x+1)) = 0$,

$-12(x^2 - 1)(x+1) = 0$,

$-12(x-1)(x+1)^2 = 0$,

$\therefore x = 1$ and $y = 8$,

or $x = -1$ and $y = -8$.

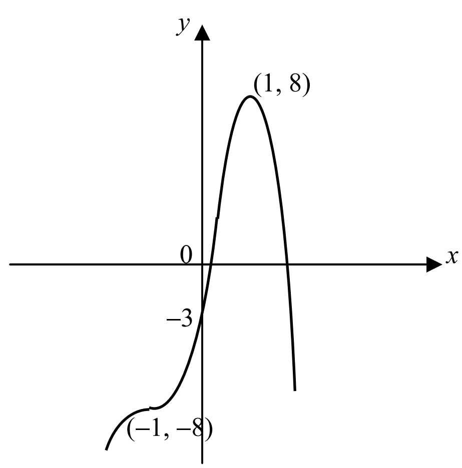
Stationary points are $(-1,-8)$ and $(1,8)$.

9b. Check the gradients on both sides of stationary points:

x	-2	-1	0	1	2
$f'(x)$	+	0	+	0	–

$\therefore (-1,-8)$ is an inflection point and $(1,8)$ is a local maximum point.

9c.



10a. $f(x) = \log_2 x = \frac{\log_e x}{\log_e 2}$, $f'(x) = \frac{1}{\log_e 2} \times \frac{1}{x} = \frac{1}{x\log_e 2}$.

10b. $f(a+h) \approx f(a) + h \times f'(a)$, let $a = 18$ and $h = 1$.

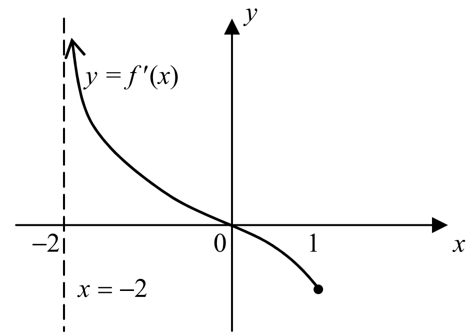
$\therefore f(19) \approx f(18) + 1 \times f'(18)$, i.e. $\log_2 19 \approx \log_2 18 + \frac{1}{18\log_e 2}$.

$\therefore \log_2 19 \approx 4.17 + \frac{1}{18 \times 0.69} = 4.17 + 0.08 = 4.25$

11a. Equation of circle: $x^2 + y^2 = 2^2$, $\therefore y = \sqrt{4 - x^2}$.

$f : [-2,1] \to R, f(x) = \sqrt{4 - x^2}$.

11b.



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