2003 VCAA Mathematical Methods Exam 1 Solutions
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Part I

1	2	3	4	5	6	7	8	9
D	B	B	D	E	A	C	D	D

10	11	12	13	14	15	16	17	18
C	C	A	A	E	E	C	E	A

19	20	21	22	23	24	25	26	27
E	C	A	E	D	D	B	B	B

Q1 Turning point at $x = 2$ on the x-axis, therefore either D or E, D gives the required section.

Q2 B is not true, $\because \log_2(2) = 1$.

Q3 $f(x) = \frac{x^3 + 1}{x} = x^2 + \frac{1}{x}$, B

Q4 D $+P$ moves e^{kx} up by P units. e^{kx} has the same shape as e^x for positive k.

Q5 $y = \sqrt{-x}$ is the reflection of $y = \sqrt{x}$ in the y-axis, $y = 2\sqrt{-x}$ is the dilation of $y = \sqrt{-x}$ away from the x-axis by 2 units. E

Q6 Since $-0.5 \le 0.5\cos(2x) \le 0.5$, therefore no solution for x. A

Q7 The function completes a cycle in π. C

Q8 $Q = 1$, $P = -2$, $4 = \frac{2\pi}{k\pi}$, $\therefore k = \frac{1}{2}$. D

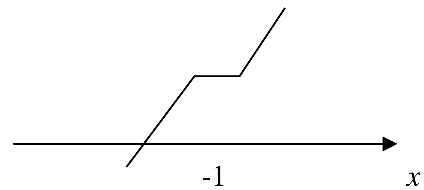
Q9 $\frac{dy}{dx} = 2 \times 2\sec^2(2x) = \frac{4}{\cos^2(2x)}$. D

Q10 $\frac{dy}{dx} = u'(x)\log_e(x) + u(x) \times \frac{1}{x}$

When $x = 2$, $\frac{dy}{dx} = u'(2)\log_e(2) + \frac{u(2)}{2}$

C

Q11 C



Q12 Since $f(x) = g'(x)$, $f(x)$ is the gradient function of g and it is negative on the interval (a, b).

A

Q13 $f(x) = \int (4e^{2x})dx = \frac{4e^{2x}}{2} + c$

$= 2e^{2x} + c$ A

Q14 $2\int_1^4 f(x)dx + \int_1^4 3dx = 2 \times 2 + [3x]_1^4$

$= 4 + 12 - 3 = 13$ E

Q15 $\int_0^5 g(x)dx = \int_0^5 f'(x)dx = [f(x)]_0^5 = f(5) - f(0)$

E.

Q16 $4 \times \int_0^{\frac{\pi}{2}} \sin(2x)dx = 4 \times \left[-\frac{\cos(2x)}{2}\right]_0^{\frac{\pi}{2}}$

$= 4 \times \left[\frac{1}{2} + \frac{1}{2}\right] = 4$ C

Q17 When $x = 0$, $y = \sqrt{3}$

$x = 1$, $y = \sqrt{2}$

$x = 2$, $y = 1$

$A = \sqrt{3} \times 1 + \sqrt{2} \times 1 + 1 \times 1$

$= \sqrt{3} + \sqrt{2} + 1$ E

Q18 $f(x)=2(x+3)^2-8$, the turning point is $(-3,-8)$. A

Q19 The term with x^2 is $10(2x)^2(-3)^3=-1080x^2$. E

Q20 Two linear factors only, $\therefore$ two real solutions. C

Q21 For f, $x=3$, $y=1$.
$\therefore$ for f^{-1}, $y=3$, $x=1$ A

Q22 $2\log_e(x)-\log_e(x+2)-\log_e(y)=1$

$$\log_e\left(\frac{x^2}{(x+2)y}\right)=1$$

$$\frac{x^2}{(x+2)y}=e^1,\ \therefore y=\frac{x^2}{e(x+2)}$$ E

Q23 Same μ because they have the same center. X_2 has a wider spread than X_1, $\therefore \sigma_1<\sigma_2$. D

Q24 Not I, because $\sum \Pr>1$.
Not IV, because probability value cannot be negative. D

Q25 $Var(X)=E(X^2)-\mu^2$

$$\mu=E(X)=0\times p+1\times(1-p)=1-p$$
$$E(X^2)=0^2\times p+1^2\times(1-p)=1-p$$
$$\sigma=\sqrt{Var(X)}=\sqrt{(1-p)-(1-p)^2}$$
$$=\sqrt{p(1-p)}$$ B

Alternatively, it is a binomial distribution with $n=1$, probability of success $=1-p$ and probability of failure $=p$. $\sigma=\sqrt{p(1-p)}$.

Q26 Because of the large population, use binomial to approximate the hypergeometric distribution.

$n=20$, $p=0.6$,
$\Pr(X=12)={}^{20}C_{12}(0.6)^{12}\times(0.4)^8$ B

Q27 Hypergeometric distribution.

$$\Pr(X\ge 1)=1-\Pr(X=0)$$
$$=1-\frac{{}^8C_2\times{}^4C_0}{{}^{12}C_2}$$
$$=1-\frac{{}^8C_2}{{}^{12}C_2}$$ B

Part II

Q1 Let $P(x)=2x^4-3x^3+7x+11$
Remainder $=P(-1)=2(-1)^4-3(-1)^3+7(-1)+11$
$\neq 0$
Therefore $P(x)$ is not exactly divisible by $(x+1)$.

Q2a Gradient of the tangent is the same as that of the given line, i.e. 3.

$$\therefore \frac{dy}{dx}=2x-2=3,\ x=\frac{5}{2},$$
$$y=\left(\frac{5}{2}\right)^2-2\left(\frac{5}{2}\right)-1=\frac{25}{4}-5-1=\frac{1}{4}$$
P is $\left(\frac{5}{2},\frac{1}{4}\right)$.

Q2b Gradient of the normal $=-\frac{1}{3}$.

Equation of normal is:
$$y-\frac{1}{4}=-\frac{1}{3}\left(x-\frac{5}{2}\right),$$
$$y=-\frac{1}{3}x+\frac{13}{12}.$$

Q3 Divide both sides by $\cos(2\pi x)$ for $\cos(2\pi x)\neq 0$.
$\tan(2\pi x)=-\sqrt{3}$, $0\le x\le 1$ i.e. $0\le 2\pi x\le 2\pi$
$2\pi x=\frac{2\pi}{3},\frac{5\pi}{3}$, $\therefore x=\frac{1}{3},\frac{5}{6}$.

Q4a x-intercept: let $2\log_e(x+3)+1=0$,

$$\log_e(x+3)=-\frac{1}{2}$$

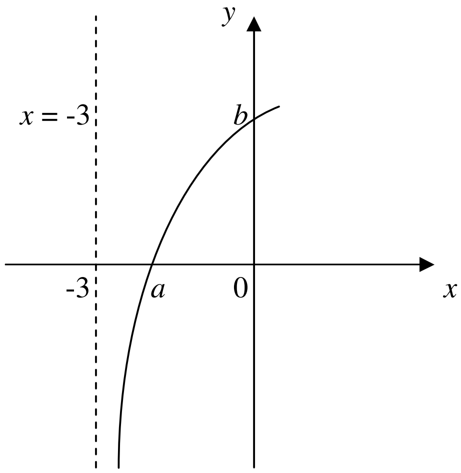
$$x+3=e^{-\frac{1}{2}}$$

$$x=e^{-\frac{1}{2}}-3$$

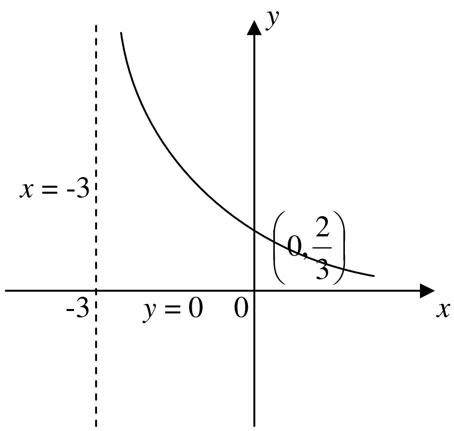
y-intercept: let $x=0$, $y=2\log_e(3)+1$

$\therefore a=e^{-\frac{1}{2}}-3$ and $b=2\log_e(3)+1$

Q4b



Q4c



Q5a Graphics calculator, $x=0.567$

Q5b Shaded area $=\int_0^{0.567}\left(-x+e^{-x}\right)dx$

$$=\left[-\frac{x^2}{2}-e^{-x}\right]_0^{0.567}$$

$$=0.27$$

Q6a Small: $\Pr(x<10)=0.0228$

Medium: $\Pr(10<x<30)=0.9545$

Large: $\Pr(x>30)=1-0.0228-0.9545$

$=0.0227$

Q6b Expected cost per plant

$=1.50\times0.0228+2.50\times0.9545+4.00\times0.0227$

$=2.51$

Expected cost for 100 plants = 251 dollars.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

