2004 VCAA Math Methods Exam 2 Solutions

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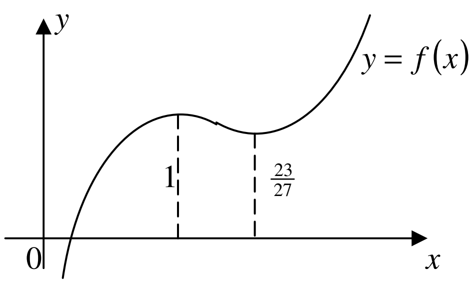
Q1a $f'(x) = 2(x-1)(x-2) + (x-1)^2$
$= (x-1)[2(x-2) + (x-1)] = (x-1)(3x-5)$
$\therefore\ u = 3,\ v = -5$

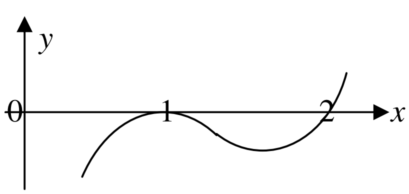
Q1b At the turning points $f'(x) = 0$,
$(x-1)(3x-5) = 0$, $\therefore x = 1$ or $\frac{5}{3}$.
When $x = 1, y = 1$ $\therefore$ one of the turning points is $(1,1)$,
$\therefore\ a = 1$ and $b = \frac{5}{3}$.

Q1c



$f(x) = p$, i.e. $f(x) - p = 0$.
For $f(x) - p = 0$ to have exactly one solution, $y = f(x)$ must be translated downwards by less than $\frac{23}{27}$ or more than 1, i.e. $p < \frac{23}{27}$ or $p > 1$.

Q1d $y = f(x) - 1 = (x-1)^2(x-2) = x^3 - 4x^2 + 5x - 2$

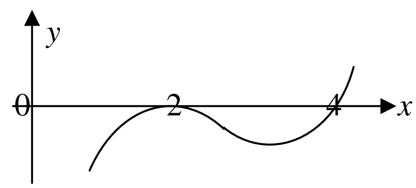


Area $= -\int_1^2 (x^3 - 4x^2 + 5x - 2)dx$
$= -\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{5x^2}{2} - 2x\right]_1^2 = 0.083$

Q1ei Dilation from the y-axis by a scale factor of 2, then a translation parallel to the y-axis by -1.

Q1eii $y = f(x) = (x-1)^2(x-2) + 1$
$y = f\left(\frac{x}{2}\right) = \left(\frac{x}{2} - 1\right)^2\left(\frac{x}{2} - 2\right) + 1$
$y = f\left(\frac{x}{2}\right) - 1 = \left(\frac{x}{2} - 1\right)^2\left(\frac{x}{2} - 2\right)$. To find the x-intercepts of the graph of the last equation, let $y = 0$,
$\therefore \left(\frac{x}{2} - 1\right)^2\left(\frac{x}{2} - 2\right) = 0$, $\therefore$ $\frac{x}{2} - 1 = 0$ or $\frac{x}{2} - 2 = 0$,
hence $x = 2$ or 4.

Q1eiii



The region is stretched horizontally by a factor of 2,
$\therefore$ area = $2 \times 0.083 = 0.17$

Q1f $f(x+h) = 1$, $\therefore$ $(x+h-1)^2(x+h-2) + 1 = 1$ or
$(x+h-1)^2(x+h-2) = 0$.
Hence $x + h - 1 = 0$, i.e. $x = 1 - h$ or $x + h - 2 = 0$, i.e. $x = 2 - h$.
To have only one of the solutions positive,
$1 - h \le 0$ and $2 - h > 0$, i.e. $1 \le h$ and $h < 2$,
$\therefore\ 1 \le h < 2$.

Q2ai $(4k^2 + 4k) + (5k^2 + 2k) + (k^2 + k) + 2k = 1$, $k > 0$
$\therefore\ 10k^2 + 9k - 1 = 0$, $k > 0$.

Q2aii $(10k - 1)(k + 1) = 0$, since $k > 0$, $\therefore$ $10k - 1 = 0$,
i.e. $k = \frac{1}{10}$.

Q2bi Binomial, $n = 9$, $p = 0.14$
$\mu = np = 9 \times 0.14 = 1.26$

Q2bii $\Pr(X = 2) = binompdf(9, 0.14, 2) = 0.245$

Q2biii $(1-0.14)^n \le 0.09$, $0.86^n \le 0.09$,
$\log_e 0.86^n \le \log_e 0.09$, $n\log_e 0.86 \le \log_e 0.09$
$n \ge \dfrac{\log_e 0.09}{\log_e 0.86} = 15.965$. The smallest whole number n greater than or equal to 15.965 is 16.

Q2c Normal, $\mu = 125$, σ?
$\Pr(X < 100) = 0.078$, $\Pr\left(Z < \dfrac{100-125}{\sigma}\right) = 0.078$,
$\therefore \dfrac{-25}{\sigma} = invNorm(0.078) = -1.4187$, $\sigma = 18$

Q2d Hypergeometric, $N = 15$, $D = 4$, $n = 3$.
$\Pr(X = 0) = \dfrac{{}^4C_0 \times {}^{11}C_3}{{}^{15}C_3}$, $\Pr(X \ge 1) = 1 - \Pr(X = 0)$
$= 0.637$

Q2e Small sample from a large population, $\therefore$ binomial approximation, $n = 12$, $p = 0.08$.
$\Pr(X \le 1) = binomcdf(12,0.08,1) = 0.751$

Q3 $P(t) = P_o e^{G(t)}$ thousand millions.
At the beginning of 1990, $t = 0$;
1991, $t = 1$ etc.
$G'(t) = a + bt,\ a,b \in R$

Q3ai $G'(t) \ge 0$, $a + bt \ge 0$ where $a > 0$ and $b < 0$,
$\therefore bt \ge -a$, $t \le -\dfrac{a}{b}$. Number of years $= -\dfrac{a}{b} - 0 = -\dfrac{a}{b}$.

Q3aii $P(t) = P_o e^{G(t)}$, $P(0) = P_o e^{G(0)}$, $P_o = P_o e^{G(0)}$,
$\therefore e^{G(0)} = 1$, $\therefore G(0) = 0$.

Q3aiii $G'(t) = a + bt$, $\therefore G(t) = at + \dfrac{1}{2}bt^2 + c$,
$G(0) = a(0) + \dfrac{1}{2}b(0)^2 + c = 0$, $\therefore c = 0$ and
$\therefore G(t) = at + \dfrac{1}{2}bt^2$.

Q3bi When $a = 0.02$ and $b = -0.0002$,
$G(t) = 0.02t - 0.0001t^2$ and $\therefore P(t) = 3e^{(0.02t - 0.0001t^2)}$

Q3bii Greatest number of years
$= -\dfrac{a}{b} = -\dfrac{0.02}{-0.0002} = 100$.

Q3biii Rate of change
$= \dfrac{dP}{dt} = 3(0.02 - 0.0002t)e^{(0.02t - 0.0001t^2)}$.

Q3biv At the beginning of 2010, $t = 20$,
$\dfrac{dP}{dt} = 3(0.02 - 0.0002 \times 20)e^{(0.02\times 20 - 0.0001\times 20^2)} = 0.069$

Q3ci $P = 3e^{0.01t}$ in thousand millions, $\dfrac{P}{3} = e^{0.01t}$,
$0.01t = \log_e\left(\dfrac{P}{3}\right)$, $\therefore t = 100\log_e\left(\dfrac{P}{3}\right)$.

Q3cii When $t = 0$, $P = 3$. Double the population, i.e. $P = 6$, t?
$t = 100\log_e\left(\dfrac{6}{3}\right) = 69.3$. The year is 1990 + 69 = 2059.

Q4a Since $-1 \le \sin\left(\dfrac{(5t-1)\pi}{2}\right) \le 1$
$\therefore$ max height = 62 + 60 = 122

Q4b min height = 62 – 60 = 2

Q4c Period $T = \dfrac{2\pi}{n} = \dfrac{2\pi}{\frac{5\pi}{2}} = 0.8$ hour i.e. 48 minutes.
$\therefore$ At 1.48 pm.

Q4di $92 = 62 + 60\sin\left(\dfrac{(5t-1)\pi}{2}\right)$,
$\therefore \sin\left(\dfrac{(5t-1)\pi}{2}\right) = \dfrac{1}{2}$, $\dfrac{(5t-1)\pi}{2} = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$.
Hence $t = \dfrac{4}{15}$ (first time), $\dfrac{8}{15}$ (second time).
$t = \dfrac{4}{15}$ hour $= 16$ minutes, $\therefore$ at 1.16 pm.

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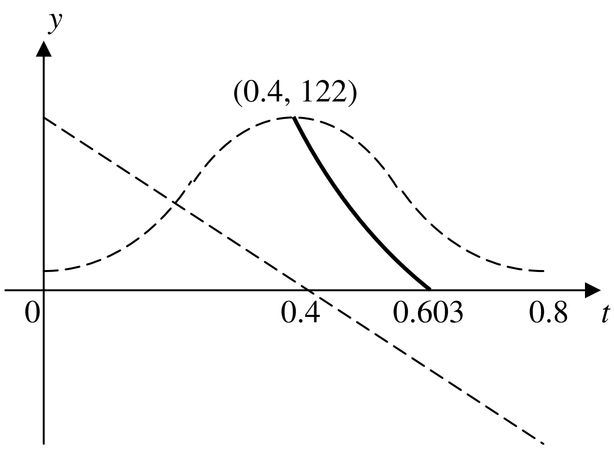
Q4dii At least 92 metres above ground level when $\frac{4}{15} \le t \le \frac{8}{15}$, $\therefore \Delta t = \frac{8}{15} - \frac{4}{15} = \frac{4}{15}$ hour, i.e. 16 minutes.

Q4ei

$$\frac{dh}{dt} = 60 \times \frac{5\pi}{2}\cos\left(\frac{(5t-1)\pi}{2}\right) = 150\pi\cos\left(\frac{(5t-1)\pi}{2}\right).$$

Q4eii When $t = 1, \frac{dh}{dt} = 150\pi\cos(2\pi) = 471.2$ mh^{-1}.

Q4f i and ii



Domain for $s(t)$ is $[0.4, 0.603]$.

Graphics calculator

Q4fiii Spider reaches ground when $s(t) = 0$, at $t = 0.603$ hour. At the highest point $t = 0.4$ hour. $\therefore \Delta t = 0.603 - 0.4 = 0.203$ hour, i.e. 12 minutes.

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