Q1a $\sqrt{\pi^2+5} \approx 3.86$ by calculator.

Q1b $2x-5>-3$, $2x>2$, $x>1$.

Q1c $\frac{1}{\sqrt{3}-1}=\frac{1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\sqrt{3}+1}{2}$.

Q1d $\frac{3}{4}+\frac{3}{16}+\frac{3}{64}+\ldots=\frac{\frac{3}{4}}{1-\frac{1}{4}}=\frac{\frac{3}{4}}{\frac{3}{4}}=1$.

Q1e $2x^2+5x-12=(2x)x+5x+(-3)(4)=(2x-3)(x+4)$.

Q1f $2x+y+4=0$, $y=-2x-4$, $m_1=-2$, $\therefore m_2=-\frac{1}{m_1}=\frac{1}{2}$.

Equation of line:
$y-3=\frac{1}{2}\left(x-{}^{-}1\right)$, $y-3=\frac{1}{2}(x+1)$, $y=\frac{1}{2}x+\frac{7}{2}$.

Q2ai $\frac{d}{dx}\left(\frac{2x}{e^x+1}\right)=\frac{2\left(e^x+1\right)-2x\left(e^x\right)}{\left(e^x+1\right)^2}=\frac{2\left(e^x-xe^x+1\right)}{\left(e^x+1\right)^2}$.

Q2aii $\frac{d}{dx}\left[(1+\tan x)^{10}\right]=10(1+\tan x)^9\frac{d}{dx}(1+\tan x)$
$=10(1+\tan x)^9\sec^2 x$.

Q2bi $\int(1+\cos 3x)dx=x+\frac{1}{3}\sin 3x+c$.

Q2bii $\int_1^4\frac{8}{x^2}dx=\int_1^4 8x^{-2}dx=\left[-8x^{-1}\right]_1^4=\left[\frac{-8}{x}\right]_1^4=-2-(-8)=6$.

Q2c $y=x\sin x$, $y'(x)=\sin x+x\cos x$,
$y'(\pi)=\sin\pi+\pi\cos\pi=-\pi$.
Equation of tangent: $y-0=-\pi(x-\pi)$, $y=-\pi x+\pi^2$.

Q3ai $AC=\sqrt{(10-2)^2+(5-11)^2}=\sqrt{8^2+(-6)^2}=10$.

Q3aii Coordinates of midpoint $=\left(\frac{10+2}{2},\frac{5+11}{2}\right)=(6,8)$.

Q3aiii $m_{OB}=\frac{16}{12}=\frac{4}{3}$, $m_{AC}=\frac{5-11}{10-2}=\frac{-6}{8}=\frac{-3}{4}$.
$m_{OB}\times m_{AC}=-1$, $\therefore OB\perp AC$.

Q3aiv Coordinates of midpoint of $OB=(6,8)$. $\therefore$ midpoints of OB and AC are the same. Hence OB and AC are perpendicular bisector of each other and $\therefore OABC$ is a rhombus.

Q3av
Hence area of $OABC=\frac{1}{2}\times OB\times AC=\frac{1}{2}\times 20\times 10=100$ sq units

Q3bi $d_n=750+100(n-1)$, where d_n (metres) is the distance she swims on the nth day.

Q3bii When $n=10$, $d=750+900=1650$.

Q3biii Total distance (metres) in the first 10 days
$S_{10}=\frac{10}{2}(750+1650)=12000$. Note: $S_n=\frac{n}{2}(a+l)$.

Q3biv $S_n=\frac{n}{2}\left[2\times 750+100(n-1)\right]$,

$34000=\frac{n}{2}\left[1500+100(n-1)\right]$, $n^2+14n-680=0$, $n=20$ days.

Q4a $\sqrt{2}\sin x=1$, $\sin x=\frac{1}{\sqrt{2}}$, $x=\frac{\pi}{4}$, $\frac{3\pi}{4}$.

Q4bi $\Pr(10)=\Pr(4+6)+\Pr(5+5)+\Pr(6+4)=3\times\frac{1}{36}=\frac{1}{12}$.

Q4bii $\Pr(10')=1-\Pr(10)=\frac{11}{12}$.

Q4ci $OA^2+AC^2=1+3=4$, $OC^2=4$, $\therefore OA^2+AC^2=OC^2$.
Hence $\angle OAC=\frac{\pi}{2}$.

Q4cii $\angle ACO=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$, $\therefore \angle AOC=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$.

Q4ciii Area of $\Delta OAC=\frac{1}{2}\times 1\times\sqrt{3}=\frac{\sqrt{3}}{2}$ m^2,

$\therefore$ area of quadrilateral $AOBC=2\times\frac{\sqrt{3}}{2}=\sqrt{3}$ m^2.

Q4civ Reflex $\angle ACB=2\pi-2\times\frac{\pi}{6}=\frac{5\pi}{3}$.

Area of major sector $ACB=\frac{\frac{5\pi}{3}}{2\pi}\times\pi\left(\sqrt{3}\right)^2=\frac{5\pi}{2}$ m^2.

Q4cv Reflex $\angle AOB=2\pi-2\times\frac{\pi}{3}=\frac{4\pi}{3}$.

Area of major sector $AOB=\frac{\frac{4\pi}{3}}{2\pi}\times\pi 1^2=\frac{2\pi}{3}$ m^2.

Total area of logo $=\frac{2\pi}{3}+\sqrt{3}+\frac{5\pi}{2}=\frac{19\pi}{6}+\sqrt{3}$ m^2.

Q5ai $\angle ABC = \frac{1}{5}(5\times 180 - 360) = 108°$.

Q5aii $BA = BC$. $\therefore \angle BAC = \angle BCA = \frac{180-108}{2} = 36°$.

Q5aiii $\angle ABF = 108° - \angle CBD = 108° - \angle BAC = 72°$.
$\therefore \angle AFB = 180 - 36 - 72 = 72°$. $\therefore \Delta ABF$ is isosceles.

Q5bi $v = \frac{2t}{16+t^2}$, $t \ge 0$. When $t=0$, $v=0$.

Q5bii $a = \frac{dv}{dt} = \frac{(16+t^2)2 - 2t(2t)}{(16+t^2)^2} = \frac{2(16-t^2)}{(16+t^2)^2}$.

Q5biii $a=0$ when $16-t^2=0$, i.e. $t=4$.

Q5biv

During the time interval $[0,4]$, displacement $s = \int_0^4 \frac{2t}{16+t^2}dt$.

Let $u = 16+t^2$. $du = 2tdt$. When $t=0$, $u=16$; when $t=4$, $u=32$.

$s = \int_{16}^{32} \frac{1}{u}du = [\log_e u]_{16}^{32} = \log_e 32 - \log_e 16 = \log_e\left(\frac{32}{16}\right) = \log_e 2$.

The position is $0 + \log_e 2 = {}^{+}\log_e 2$ metres.

Q6a $2e^{2x} - e^x = 0$, $e^x(2e^x - 1) = 0$. Since $e^x > 0$,
$\therefore 2e^x - 1 = 0$, $e^x = \frac{1}{2}$, $x = \log_e\left(\frac{1}{2}\right) = -\log_e 2$.

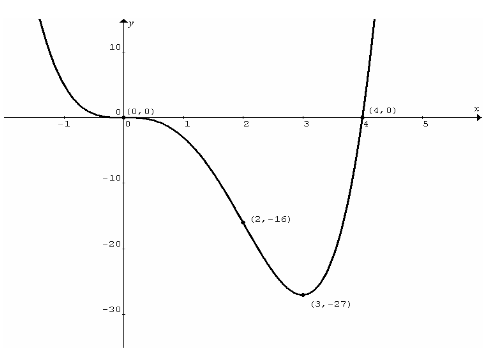
Q6bi x-intercepts: Let $x^4 - 4x^3 = 0$. $x^3(x-4) = 0$.
$(0,0)$ and $(4,0)$.

Q6bii Stationary points: $f'(x) = 0$, $4x^3 - 12x^2 = 0$,
$4x^2(x-3) = 0$. $\therefore x = 0$ or 3. When $x=3$, $y=-27$.
$(0,0)$ and $(3,-27)$.

	−1	0	1	3	4
$f'(x)$	–	0	–	0	+
Nature		Inflection point		Local minimum	

Q6biii Inflection points: $f''(x) = 0$, $12x^2 - 24x = 0$,
$12x(x-2) = 0$. $\therefore x = 0$ or 2. When $x=2$, $y=-16$.
$(0,0)$ and $(2,-16)$.

Q6biv



Q7ai $y = x^2 + 4$, $4a(y-4) = x^2$, $a = \frac{1}{4}$. Focus S is $\left(0, \frac{17}{4}\right)$.

Q7aii $x^2 + 4 = x + k$, $\therefore x^2 - x + 4 - k = 0$.

Q7aiii One intersection only, $\Delta = 0$, $(-1)^2 - 4(1)(4-k) = 0$,
$k = \frac{15}{4}$.

Q7aiv $\therefore x^2 - x + 4 - \frac{15}{4} = 0$, $x^2 - x + \frac{1}{4} = 0$, $\left(x - \frac{1}{2}\right)^2 = 0$,

$\therefore x = \frac{1}{2}$ and $y = x + k = \frac{1}{2} + \frac{15}{4} = \frac{17}{4}$. P is $\left(\frac{1}{2}, \frac{17}{4}\right)$.

Q7av S and P have the same y-coordinate, $\therefore SP$ is horizontal. The directrix of the parabola is also horizontal. $\therefore SP$ is parallel to the directrix.

Q7bi $\sqrt{3}\cos x = \sin x$, $\therefore \frac{\sin x}{\cos x} = \sqrt{3}$, $\tan x = \sqrt{3}$,

$x = \frac{\pi}{3}$ for A, $\frac{4\pi}{3}$ for B.

Q7bii Area $= \int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} (\sin x - \sqrt{3}\cos x)dx = [-\cos x - \sqrt{3}\sin x]_{\frac{\pi}{3}}^{\frac{4\pi}{3}}$

$= -\cos\frac{4\pi}{3} - \sqrt{3}\sin\frac{4\pi}{3} + \cos\frac{\pi}{3} + \sqrt{3}\sin\frac{\pi}{3}$

$= \frac{1}{2} + \frac{3}{2} + \frac{1}{2} + \frac{3}{2} = 4$ sq units.

Q8ai $1600 = Ae^k$ ……(1) and $2600 = Ae^{2k}$ ……(2)
$\frac{(2)}{(1)}$, $\frac{2600}{1600} = \frac{Ae^{2k}}{Ae^k}$, $\therefore \frac{13}{8} = e^k$, $k = \log_e\left(\frac{13}{8}\right)$ and
$A = \frac{1600}{e^k} = \frac{12800}{13}$.

Q8aii $Ae^{kt} = 4000$, $t = \frac{1}{k}\log_e\left(\frac{4000}{A}\right)$, $t \approx 2.8873$, or 2 years and 10.6 months, i.e. during October 2010.

Q8bi Given $AE // BD$, $\angle EAB = \angle DBC$ (corresponding angles), and given $\angle ABE = \angle BCD$, $\therefore \angle AEB = \angle BDC$.
$\therefore \Delta ABE ||| \Delta BCD$.

Q8bii Given $AE // BD$, $\angle AEB = \angle EBD$ (alternate angles), and given $\angle ABE = \angle BDE$, $\therefore \angle BAE = \angle BED$.
$\therefore \Delta ABE ||| \Delta EDB$. From Q8bi, $\Delta EDB || \Delta BCD$.

Q8biii Since ΔBCD, ΔEDB and ΔABE are similar, pairs of corresponding sides have the same ratio, i.e. $\frac{p}{8} = \frac{q}{p} = \frac{27}{q}$.

$\therefore 8 + p + q + 27 + ...$ form a geometric series.

Q8biv Let r be the common ratio. $\therefore 27 = 8r^3$, $r = \frac{3}{2}$.

$\therefore p = 8r = 12$, $q = pr = 18$.

Q9a $V=\int_0^1 \pi(x^2+1)^2\,dx=\int_0^1 \pi(x^4+2x^2+1)\,dx$

$=\left[\pi\left(\frac{x^5}{5}+\frac{2x^3}{3}+x\right)\right]_0^1=\pi\left(\frac{1}{5}+\frac{2}{3}+1\right)=\frac{28\pi}{15}$ cubic units.

Q9bi $\Pr=\frac{52}{52}\times\frac{39}{51}=\frac{39}{51}$.

Q9bii $\Pr=\frac{52}{52}\times\frac{39}{51}\times\frac{26}{50}\times\frac{13}{49}=\frac{2197}{20825}$

Q9ci $A_n=PR+PR^2+PR^3+......+PR^n=\frac{PR(R^n-1)}{R-1}$, where $R=1+0.06=1.06$. $A_{18}=\frac{1000\times1.06(1.06^{18}-1)}{1.06-1}=\$32759.99$.

Q9cii $A_3=(PR^2)R+(PR)R^2+(P)R^3=3PR^3$, where $R=1.06$.
$A_3=3\times1000\times1.06^3=\$3573.05$.

Q9ciii $A_{18}=18PR^{18}=18\times1000\times1.06^{18}=\$51378.10$.

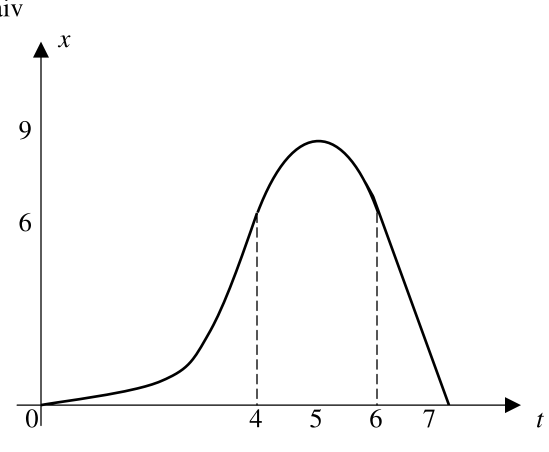
Q10ai Distance $\approx\frac{\Delta x}{3}\left(f(x_0)+4f(x_1)+f(x_2)\right)=\frac{2}{3}(0+4+5)=6$.

Q10aii $t>5$.

Q10aiii Area under graph and above t-axis
= area below t-axis and above graph . $t\approx7.6$.

Using the answer in Q10ai, $0\approx6-5(t-6)$, $t\approx7.2$.

Q10aiv



Q10bi $N=N_1+N_2=\frac{L_1}{x^2}+\frac{L_2}{(m-x)^2}$.

Q10bii Minimum N when $\frac{dN}{dx}=0$,

i.e. $\frac{-2L_1}{x^3}+\frac{2L_2}{(m-x)^3}=0$.

$\frac{L_1}{x^3}=\frac{L_2}{(m-x)^3}$, $\left(\frac{m-x}{x}\right)^3=\frac{L_2}{L_1}$,

$\frac{m}{x}=\frac{\sqrt[3]{L_2}}{\sqrt[3]{L_1}}+1$, $x=\frac{m}{1+\sqrt[3]{\frac{L_2}{L_1}}}$,

or $\frac{m}{x}=\frac{\sqrt[3]{L_2}+\sqrt[3]{L_1}}{\sqrt[3]{L_1}}$, $x=\frac{m\times\sqrt[3]{L_1}}{\sqrt[3]{L_1}+\sqrt[3]{L_2}}$.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors.