Physics notes –
Interactions of light and matter

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Light has been described both as a particle and as a wave.

Isaac Newton (~1665) made up a **particle model** of light to explain many of the known behaviours of light at that time. He was able to explain

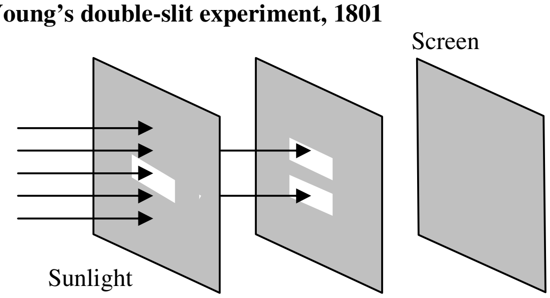
- straight line propagation of light
- the intensity of light
- the reflection of light from flat and curved surfaces
- the refraction of light as it crosses the interface between two media.

He was unable to explain

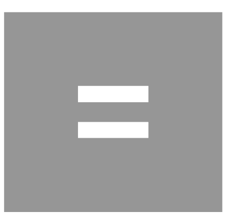
- partial reflection and partial transmission of light at an interface
- the existence of Newton's rings and other related phenomena due to the interference of light.

Christiaan Huygens (~1678) considered light as a wave. Using a **wave model** he was able to explain all the known phenomena of light mentioned above as well as **interference** and **diffraction** of light.

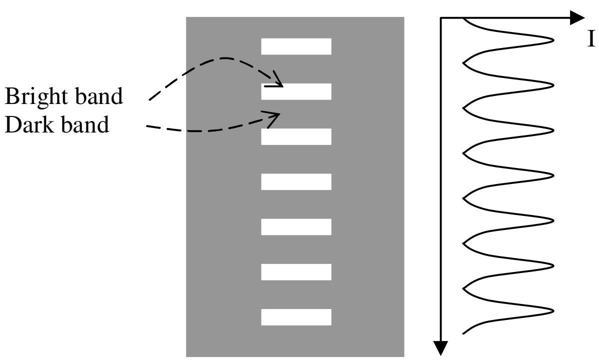
Young's double-slit experiment, 1801



If light consists of particles, we would expect to see two bright bands on the screen.



But Young observed many bright and dark bands.

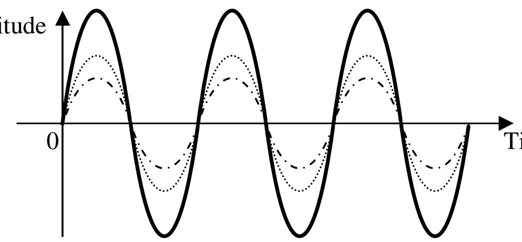


Young was able to explain this result as a wave-interference phenomenon – the double-slit **interference pattern** demonstrates the wave-like nature of light.

Explaining the interference pattern using the wave model

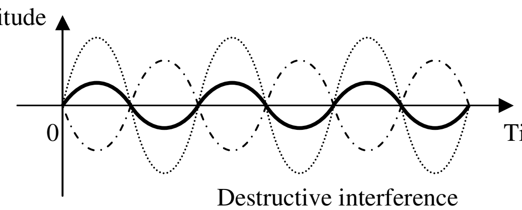
The single slit provides the double slits with **coherent** light waves (Refer to page 2).

The bright bands are formed when light waves from the two slits arrive at the screen *in phase*, i.e. wave crest combines with crest and wave trough combines with trough. This is known as **constructive interference**. The following diagram shows the sum of the two waves as a function of time at a bright band.



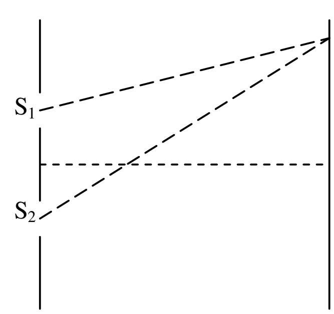
Constructive interference results in greater amplitude

The dark bands are formed when the two waves arriving at the screen are half a cycle *out of phase*, i.e. wave crest combines with wave trough. This is called **destructive interference**. The following diagram shows the sum of the two waves at a dark band.



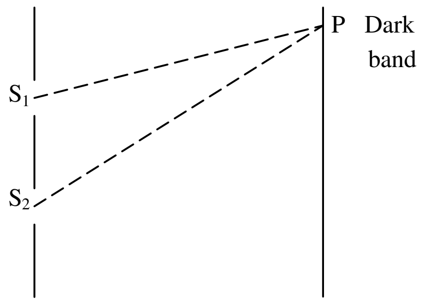
Destructive interference results in smaller amplitude

Constructive interference is possible when the difference in distances from the two slits to the screen (i.e. path difference) is zero or equal to an integral multiple of the wavelength.



Path difference $pd = n\lambda$, i.e. $S_2P - S_1P = n\lambda$, where $n = 0, 1, 2, \ldots$

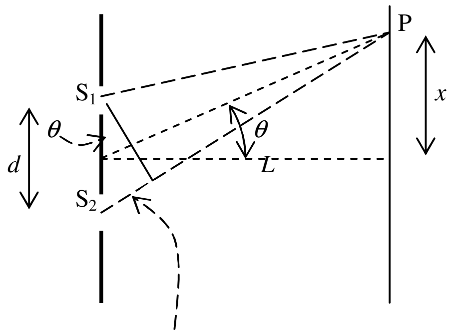
For destructive interference the path difference is an *odd* multiple of half a wavelength.



Path difference $pd = n\left(\frac{\lambda}{2}\right)$, i.e. $S_2P - S_1P = n\left(\frac{\lambda}{2}\right)$, where n = 1, 3, 5,.....

Alternatively, $pd = \left(n - \frac{1}{2}\right)\lambda$, n = 1, 2, 3……

The path difference can be calculated if the separation d between the slits and θ are known.



Path difference $pd = d\sin\theta$

$\therefore d\sin\theta = n\lambda$, n = 0, 1, 2, ... for bright bands, and

$d\sin\theta = \left(n - \frac{1}{2}\right)\lambda$, n = 1, 2, 3, ... for dark bands.

Effect of wavelength, distance of screen and slit separation on interference patterns

For small θ, $\sin\theta \approx \tan\theta = \frac{x}{L}$.

$\therefore d\sin\theta \approx d\frac{x}{L} = n\lambda$, $\therefore d\frac{\Delta x}{L} = \lambda$, $\Delta x = \frac{L\lambda}{d}$.

The last equation shows that the separation between two adjacent bright (or dark) bands Δx increases as L or λ increases, or as d decreases.

Coherent and incoherent sources

At high temperature (>1000 K) objects glow, e.g. the Sun, light bulbs, electric stove burners etc. The light emitted from these objects contains a wide spectrum of frequencies. In a heated object the electrons absorb thermal energy and move to different excited states, they then return to the stable state by emitting lights of different frequencies. Due to the difference in frequencies, the emitted lights can never be in phase and the light source (the heated object) is said to be **incoherent**. Laser is a **coherent** source because all emissions have the same frequency and occur at the same time.

Two separate sources can also be described in terms of coherence. In Young's experiment the two slits act as if they were **coherent** sources of radiation. They are described as coherent because the light waves leaving them bear the same phase relationship to each other at all times, e.g. if a crest is leaving one slit then a crest is leaving the other slit at the same time at all times.

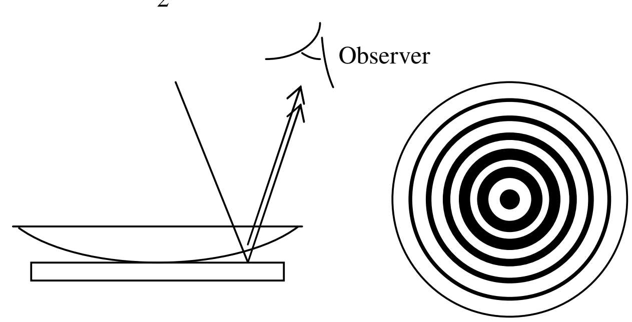
An interference pattern is possible only when the sources are relatively coherent. The following setups (modifications of Young's double-slit experiment) would not produce an interference pattern because the lights emitted from the two sources would have a random phase with respect to each other. They are **incoherent** sources.

(i) Replace the two slits with two light globes.
(ii) Use separate light globes to illuminate the slits.
(iii) The single slit is replaced with a large light globe and the light passing through the two slits comes from different parts of the filament.
(iv) Replace the two slits with two identical laser pointers.

Newton's rings

When a curved glass surface is placed in contact with a flat glass surface, a series of concentric rings (first observed by Robert Hooke) is seen when illuminated from above by monochromatic (i.e. single colour) light. They are called Newton's rings (named after Newton because Newton gave an elaborate description of them). These rings are formed when rays reflected by the top and bottom surfaces of the air gap between the two pieces of glass interfere.

Bright rings are formed due to constructive interference when the path difference between the two rays is a multiple of λ. Dark rings – destructive interference – path difference is a multiple of $\frac{\lambda}{2}$.



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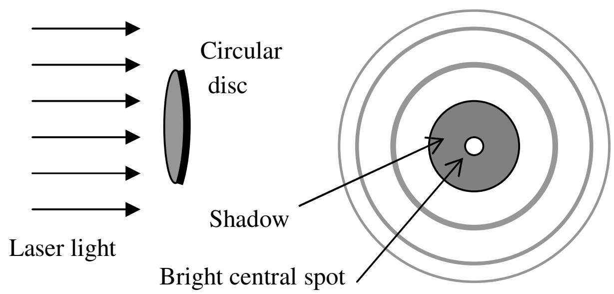
Diffraction

Diffraction is also an important property of waves. Light diffracts (changes direction of propagation) when it passes through a narrow opening or around an obstacle. This is another evidence that light behaves like a wave. Full acceptance of the wave model came only with studies on diffraction of light more than a decade after Young's double-slit experiment.

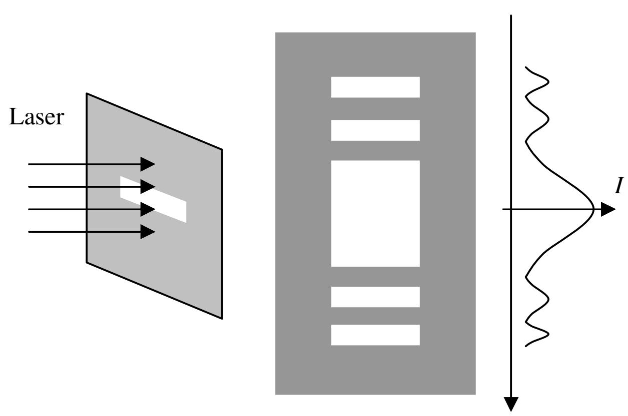
The bright and dark fringes appearing in a diffraction pattern are caused by the interference of the diffracted light waves.

Diffraction by an obstacle

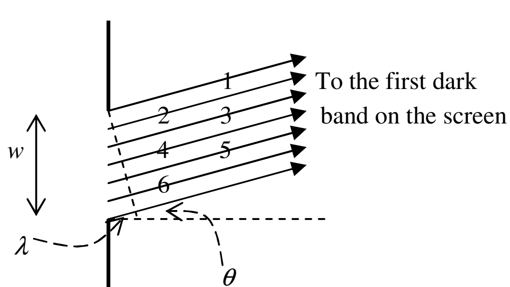


The bright central spot is the result of constructive interfercence. The shadow is cast by the obstacle. The dark circular fringes are caused by destructive interference.

Diffraction by a single slit



The wide central bright region results from the diffraction of light passing through the slit. The dark fringes are due to destructive interference of light rays from the slit.



Rays 1 and 4 have a path difference of $\frac{\lambda}{2}$, hence they result in destructive interference. Likewise, the pairs 2-5 and 3-6 form destructive interference.

For the first dark band on each side of the central bright region, $\sin\theta = \frac{\lambda}{w}$. For small θ, $\sin\theta \approx \theta$, $\therefore \theta \approx \frac{\lambda}{w}$.

The first dark band on each side of the central bright region defines the directional spread (diffraction) of the light through the slit. The wider the spread the greater the angle θ.

Hence the directional spread (diffraction) of light through a slit is related to the ratio $\frac{\lambda}{w}$.

Diffraction is significant when $\frac{\lambda}{w} \approx$ or > 1.

Directional spread (diffraction) increases when λ increases. This explains why red light diffracts more than blue. Decreasing the width of the slit also increases diffraction.

Discovery of the electron

In 1895, **J. J. Thomson** discovered the electron.

In 1896, **Millikan** verified that charges were **'quantised'** through his famous oil droplets experiment. He discovered that the droplets were charged with integral multiples of one charge, the charge of an electron.

The unit to measure the amount of charge is **Coulomb** (C).

The charge of an electron = –1.602 x 10^{-19} C.

Electron-volt, eV

Instead of joules, a more convenient energy unit for sub-atomic particles is **electron-volt**, eV.

One electron-volt is the amount of energy gained or lost by a particle of one electron charge when it moves across a potential difference of one volt.

$$\Delta E_k = qV = 1\text{e} \times 1\text{V} = 1\text{ eV}$$

$$\Delta E_k = qV = 1.602\times10^{-19}\text{ C} \times 1\text{V} = 1.602\times10^{-19}\text{ J}$$

$$\therefore\ 1\text{ eV} = 1.602\times10^{-19}\text{ J}$$

E.g., when a particle with two excess electrons on it moves from the negative to the positive plate and the potential difference between the plates is 12 V, it gains 24 eV of kinetic energy.

Example 1 What is 24 eV in joules?

$24\text{ eV} = 24\times1.602\times10^{-19}\text{ J} = 3.8\times10^{-18}$ J.

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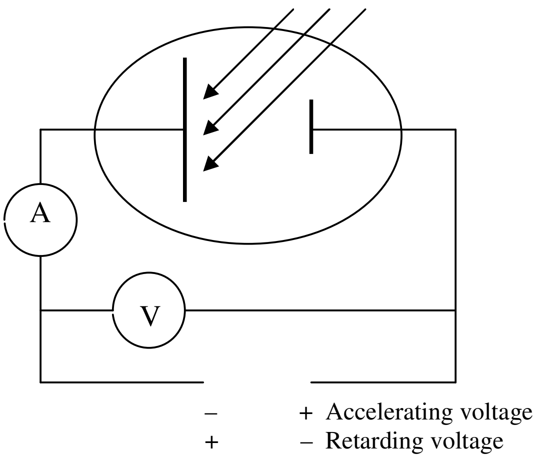
The photoelectric effect

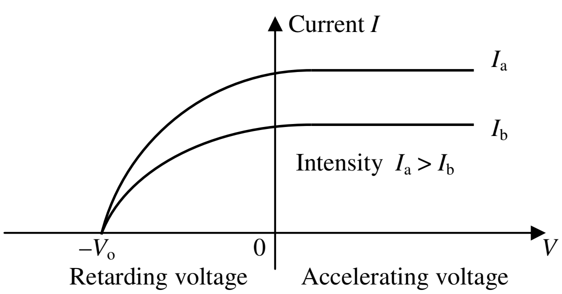
When light shines on certain materials (mainly metals), e.g. lithium, zinc etc, electrons are emitted from the surface if the right colour (frequency) of light is used. This occurrence is known as the **photoelectric effect**.

Setup used to investigate the photoelectric effect

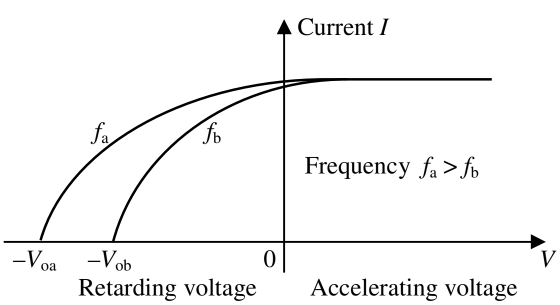


The results

I* vs *V using light of the same frequency but different intensities, I_a, I_b.

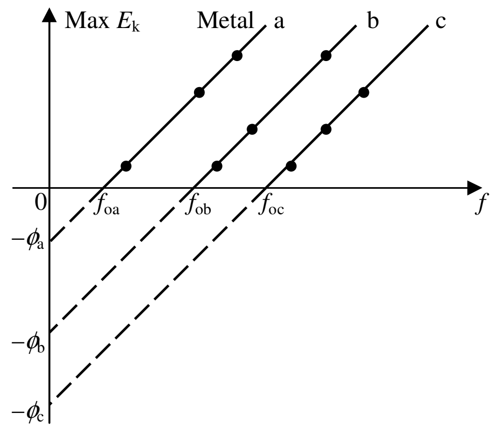


I* vs *V using light of the same intensity but different frequencies, f_a, f_b.



If V_o (in volts) is the retarding voltage (called stopping voltage) required to stop all the emitted electrons for a particular frequency, then V_o eV (or qV_o joules) is the maximum kinetic energy, $E_{k,max}$, of the electrons.
($q = 1.602 \times 10^{-19}$ C)

Maximum E_k vs f for different metals



The above graphs for different metals, *a*, *b*, *c* etc are straight lines. They are parallel, i.e. they have the same gradient *h*, which is known as **Planck's constant**.

When each line is extrapolated to the left, it cuts the vertical axis at $-\phi$.

The equation of each line is $E_{k,max} = hf - \phi$, where ϕ is the required minimum amount of energy to be absorbed by the electron in order for it to escape from the metal. It is called the **work function** of the metal.

The value of Planck's constant *h* is

$h = 6.63 \times 10^{-34}$ Js or $h = 4.14 \times 10^{-15}$ eVs.

The horizontal axis intercept is called the **threshold frequency** $\boldsymbol{f_o}$. It is the minimum frequency required for photoelectric effect to occur for a particular metal. Different metals have different threshold frequencies, e.g.

$f_o = 7.0 \times 10^{14}$Hz for silver and

$f_o = 9.0 \times 10^{14}$Hz for zinc.

The work function ϕ and the threshold frequency f_o are related,

$$\phi = hf_o .$$

Example 1 Given the threshold frequencies of silver and zinc, calculate their work functions.

For silver, $\phi = hf_o = \left(6.63\times10^{-34}\right)\left(7.0\times10^{14}\right) = 4.6\times10^{-19}$ J,
or $= \left(4.14\times10^{-15}\right)\left(7.0\times10^{14}\right) = 2.9$ eV.

For zinc, $\phi = hf_o = \left(6.63\times10^{-34}\right)\left(9.0\times10^{14}\right) = 6.0\times10^{-19}$ J,
or $= \left(4.14\times10^{-15}\right)\left(9.0\times10^{14}\right) = 3.7$ eV.

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Failure of the wave model to explain the photoelectric effect

According to the wave model, light is a continuous wave and the intensity is related to its amplitude, which measures the energy of the wave. Therefore an electron can absorb any amount of light energy, depending on the time interval it is exposed to the light wave.

The wave model failed to explain why
- the maximum kinetic energy remained the same when the intensity was changed;
- the maximum kinetic energy changed with the frequency of light used;
- there was a threshold frequency for each metal used.

Einstein's explanation of the photoelectric effect – The photon model

According to Einstein's photon model, a beam of light is a beam of particles called **photons**. Single colour light (i.e. light of a single frequency f) consists of photons of the same energy given by hf. There are more photons in a more intense beam of light.

When photons strike a metal, some will be absorbed by the electrons in the metal. To have photoelectrons emitted, the energy of each photon must be high enough in order for the electrons to overcome the bonding energy, i.e. the work function ϕ. As the photons penetrate into the metal they may collide with other electrons before they are absorbed. Each collision lowers the photon frequency slightly and hence the scattered photon has slightly lower energy. This is known as **the Compton effect** (see next section). Therefore electrons at the surface escape with maximum kinetic energy, while those inside escape with less energy or are unable to escape at all,

$$E_{k,max} = hf - \phi .$$

Example 1 Calculate the energy of a photon of ultraviolet light ($\lambda = 122$ nm).
Note: nm stands for nanometre, 1nm = 1×10^{-9} m.

Since $c = f\lambda$, $\therefore E = hf = \frac{hc}{\lambda}$

$$= \frac{(6.63\times10^{-34})(3.0\times10^{8})}{1.22\times10^{-7}} = 1.63\times10^{-18} \text{ J},$$

$$\text{or } = \frac{(4.14\times10^{-15})(3.0\times10^{8})}{1.22\times10^{-7}} = 10 \text{ eV}.$$

Example 2 Find the max. kinetic energy and speed of an electron emitted from a sodium surface ($\phi = 2.28$ eV) when illuminated by light of wavelength (a) 410 nm and (b) 550 nm. (Mass of electron $m = 9.1 \times 10^{-31}$ kg)

(a) $E_{k,max} = hf - \phi = \frac{hc}{\lambda} - \phi$

$$= \frac{(4.14\times10^{-15})(3.0\times10^{8})}{4.10\times10^{-7}} - 2.28 = 0.75 \text{ eV or } 1.2\times10^{-19} \text{ J}.$$

$$E_k = \frac{1}{2}mv^2,\ v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2(1.2\times10^{-19})}{9.1\times10^{-31}}} = 5.1\times10^{5} \text{ ms}^{-1}.$$

(b) Photon energy $\frac{hc}{\lambda} = \frac{(4.14\times10^{-15})(3.0\times10^{8})}{5.50\times10^{-7}} = 2.26$ eV.

It is less than the work function $\phi = 2.28$ eV. $\therefore$ no emission of photoelectrons.

The Compton Effect and photon momentum

The photon model was further supported by the discovery of the **Compton Effect**. In 1923, A. H. Compton scattered X rays from various materials. He found the scattered light had a slightly lower frequency than did the incident light, pointing to a loss of energy and a transfer of momentum.

The momentum of a photon is given by $p = \frac{E}{c}$.

Since $E = hf$ and $c = f\lambda$, $\therefore p = \frac{hf}{c} = \frac{h}{\lambda}$.

For light with wavelength λ, the energy and momentum of an incoming photon are $E = \frac{hc}{\lambda}$ and $p = \frac{h}{\lambda}$.

After scattering, frequency f' is lower (wavelength λ' is longer), hence $E' = hf' = \frac{hc}{\lambda'}$ and $p' = \frac{hf'}{c} = \frac{h}{\lambda'}$ are lower.

Example 1 Calculate the photon momentum of ultraviolet light.

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{1.22\times10^{-7}} = 5.4\times10^{-27} \text{ kg ms}^{-1}.$$

Alternatively, $p = \frac{E}{c} = \frac{1.63\times10^{-18}}{3.0\times10^{8}} = 5.4\times10^{-27}$ kg ms^{-1}.

Wave-particle duality

Wave model – supported by interference and diffraction.

Particle (photon) model – supported by the photoelectric effect and the Compton Effect.

These two models of light appear to be inconsistent with each other but both have been shown to be valid depending on the circumstances. Physicists have concluded that this duality of light must be accepted as the nature of light. It is referred to as the **wave-particle duality**. Light is more complex than just a simple wave or a simple beam of particles.

Wave nature of matter

In 1923 Louis de Broglie applied the idea of wave-particle duality of light to matter. He argued that matter, material particles such as electrons might also have wave-particle duality.

De Broglie proposed that the wavelength of a moving material particle would be related to its momentum in the same way as for a photon, $p = \frac{h}{\lambda}$. For a particle of mass m moving with speed v, the wavelength λ is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$.

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Since $p = \sqrt{2mE_K}$, $\therefore \lambda = \dfrac{h}{\sqrt{2mE_K}}$ when the kinetic energy of the particle is E_k .

This wavelength λ assigned to a moving particle is called the de Broglie wavelength of the particle.

The above equations are valid when m is in kg, v in ms^{-1}, E_k in J and h in Js.

Example 1 Calculate the de Broglie wavelength of a 0.10 kg object moving at 30 ms^{-1}.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}}{0.10\times30} = 2.2\times10^{-34}\ \text{m}.$$

The wavelength obtained in example 1 is incredibly small, (interatomic spacing ≈ 10^{-10} m). There are no known objects (obstacles) or slits (openings) to diffract waves of such a short wavelength, so the wave properties of ordinary objects are unobservable.

Example 2 Calculate the de Broglie wavelength of an electron accelerated from rest through a potential difference of 100 V.

Gain in $E_k = (1.602\times10^{-19})(100) = 1.602\times10^{-17}$ J.

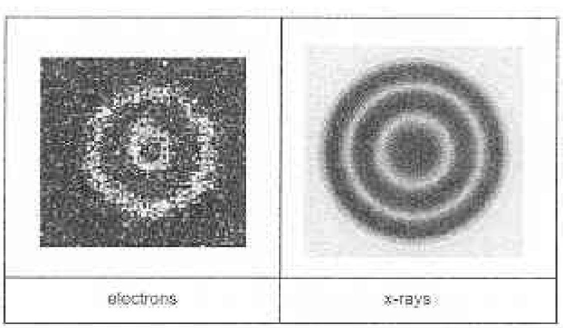
$$\therefore \lambda = \frac{h}{\sqrt{2mE_K}} = \frac{6.63\times10^{-34}}{\sqrt{2(9.1\times10^{-31})(1.602\times10^{-17})}} = 1.2\times10^{-10}\ \text{m}.$$

In example 2, the wavelength of the electron is on the order of 10^{-10} m. This wavelength is observable because the spacing of atoms in a crystal is on the same order, i.e. $\frac{\lambda}{w} \approx 1$.

Experimental confirmation of the wave nature of matter

In early 1927, C. J. Davisson and L. H. Germer observed the diffraction of electrons from the surface of a metal crystal. The diffraction pattern produced was similar to that formed by using X-rays (photons). From the scattering of the electrons they calculated the wavelength to be that predicted by de Broglie. Other experiments showed that protons, neutrons and other particles also have wave properties.

Example 1



The two images have been obtained by scattering electrons and X-rays off a collection of many small crystals with random orientations. The pictures are all to the same scale, and the X-rays have a wavelength of 3.5 × 10^{-11} m.

(a) Calculate the energy of the X-rays in keV.
(b) Estimate the de Broglie wavelength of the electrons.
(Source: Physics (Pilot) Written exams – June & Nov)
(c) What feature of the diffraction patterns is related to the fact that the target is a collection of many small crystals with random orientations?

(a) $E = \dfrac{hc}{\lambda} = \dfrac{(4.14\times10^{-15})(3.0\times10^{8})}{3.5\times10^{-11}} = 3.5\times10^{4}\ \text{eV} = 35\ \text{keV}.$

(b) The diffraction pattern produced by using electrons was similar to that formed by using X-rays. ∴ the de Broglie wavelength of the electrons ≈ 3.5 × 10^{-11} m.

(c) Instead of a single crystal, the target was a collection of many small crystals with random orientations.

Comparing the momentum of photons and of matter

Momentum of photons, $p = \dfrac{h}{\lambda}$.

Momentum of matter, $p = mv$, or $p = \dfrac{h}{\lambda}$ if the matter wavelength is known.

∴ photons and matter have the same momentum if the have the same wavelength.

Absorption and emission spectra

Diffraction causes light to spread out into its component wavelengths, the resulting pattern is called a spectrum of the light. The device used to diffract the light is called a *diffraction grating*.

When a gas is heated or a large current is passed through it, e.g. metal vapour lamps, the gas glows and emits a characteristic diffraction pattern called an *emission line spectrum*. The emission line spectrum suggests that only certain wavelengths of light are emitted. Each line in the spectrum corresponds to a particular wavelength. A different gas produces a different spectrum.

The sun is a dense gaseous object and it emits a continuous spectrum including a wide range of wavelengths. After passing through the sun's outer atmosphere and the earth's atmosphere, a number of dark lines appear in the sun's spectrum. These dark lines (called absorption lines) are caused by the absorption of certain wavelengths of the sunlight by the atoms and molecules in the atmospheres. The resulting pattern is called an *absorption line spectrum*.

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Quantised (discrete) energy levels for an atom

Emission and absorption line spectra are evidence for quantised atomic energy levels. Electrons move around a nucleus with discrete energies.

When electrons 'jump' from one orbit (energy level) to another, they gain or lose energy in discrete amounts equal to the difference between the two energy levels.

It is useful to show the various energy levels as horizontal lines presented in an energy-level diagram as shown below, e.g. energy levels for the *hydrogen atom.*

Ionisation	- - - - - - - - - - - - -	E = 0 eV
Excited states	$n = 4$	-0.85 eV
	$n = 3$	-1.5 eV
	$n = 2$	-3.4 eV
Ground state	$n = 1$	-13.6 eV

Interactions of light and matter

When a photon passes through a gas, it interacts with the atoms and their electrons. There are three main types of interactions.

(1) The photon can be scattered off an electron or a nucleus and in the process lose some energy. This is the Compton Effect discussed earlier.

(2) The photon may knock an electron out of an atom and in the process the photon is completely absorbed by the atom. The atom is said be ionised. This is the same process as the photoelectric effect.

For hydrogen gas the photon energy needs to be greater than 13.6 eV for ionisation to occur.

(3) A photon can excite an electron from the ground state to a higher energy level if its energy is not enough to ionise the atom. This is possible when the energy of the photon exactly matches the energy difference between two energy levels and in the process the photon is completely absorbed by the atom. This gives rise to the absorption spectrum.

For example, a photon of 12.1 eV energy can knock an electron in a hydrogen atom from its ground state ($n = 1$) to the second excited state ($n = 3$) .

When an electron 'jumps' to a lower level, the atom emits a photon. The emission spectrum shows the different transitions from a higher to a lower energy level, e.g. from $n = 2$ to $n = 1$, the photon energy is:
$E =$ (-3.4 eV) – (-13.6 eV) = 10.2 eV or 1.63×10^{-18} J.

The frequency of the emitted photon is:
$f = \frac{E}{h} = 2.46\times10^{15}$ Hz.

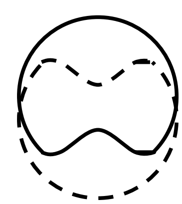
The wavelength of the emitted photon is:
$\lambda = \frac{hc}{E} = 1.22 \times 10^{-7}$ m or 122 nm, which is in the UV region.

Quantised energy levels and de Broglie's hypothesis

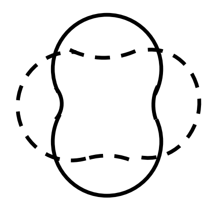
Bohr suggested the orbital structure of an atom but could not give an explanation for the quantised energies of the orbits.

Louis de Broglie used the idea of matter wave (dual nature of matter) to explain why electrons in orbits have quantised energies. According to de Broglie each electron orbit in an atom is a standing wave. The only waves that persist are those for which the circumference of the circular orbit contains an integral multiple of a wavelength. The following diagrams show the first three possibilities (lowest multiples of wavelength):

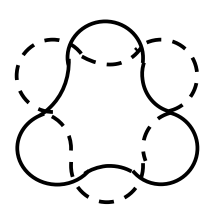
$n = 1$



$n = 2$



$n = 3$



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