Physics notes – Motion

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Scalar and vector quantities

A scalar quantity requires a numerical value and a unit to specify it, e.g. distance 6.5 km and mass 10 kg are scalar quantities.

Example 1 Name two more scalar quantities.

Surface area, e.g. 12 m^2; air pressure, e.g. 0.5 kPa

A vector quantity requires a numerical value together with a unit and a direction to specify it completely. The numerical value with the unit is called the **magnitude** of the vector quantity. Examples of vector quantities are: force, 9.8 N left; velocity, 70 km h^{-1} N35°W.

Example 2 Name two other examples of vector quantities.

Gravitational field, e.g. 9.8 N kg^{-1} downward; momentum change, e.g. 2.0 kg ms^{-1} SE

Vector quantities in one dimension

In one dimension, a positive or negative sign is used to indicate the direction of a vector quantity. Usually positive is chosen for to the right or upward direction, e.g. a force of 5 N to the left is written as $^{-}5$ N; an upward velocity of 20 ms^{-1} is written as $^{+}20$ ms^{-1}.

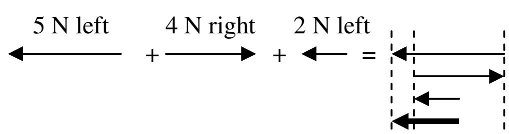
Vector quantities can also be represented by arrows drawn to scales. The length of the arrow shows the magnitude, and the direction is shown by the arrow head.

Addition of vector quantities in one dimension

Example 1 Three forces act on the same object: 5 N left, 4 N right and 2 N left. Find the **net force** on the object.

Consider vector quantities as **directed numbers**:
$^{-}5 + ^{+}4 + ^{-}2 = ^{-}3$ N, i.e. 3 N left.

Graphically: When vector quantities are represented by arrows, addition is done by placing the head of the second arrow to the tail of the first arrow. This is repeated if more than two arrows are involved. The **resultant** is an arrow starting from the tail of the last arrow to the head of the first.



Resultant (net force) is 3 N left.

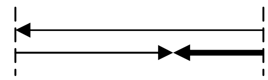
Note: The order that this is carried out does not affect the resultant.

Example 2 A car travels to the west for 1.2 km and then to the east for 0.7 km. Find the position of the car from its starting point.

As directed numbers: Take east as the positive direction.

$^{-}1.2 + ^{+}0.7 = ^{-}0.5$, i.e. 0.5 km west of the starting point.

Graphically:



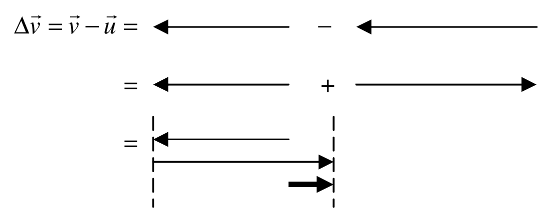
Subtraction of vector quantities in one dimension

Example 3 The velocity of a car is reduced from 75 km h^{-1} west to 60 km h^{-1} west. What is the change in velocity of the car?

As directed numbers: Take east as the positive direction.

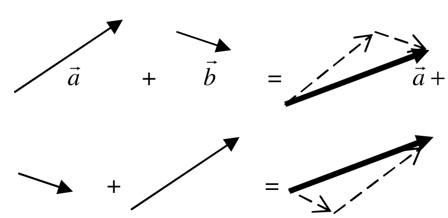
$\Delta\vec{v} = \vec{v} - \vec{u} = {}^{-}60 - {}^{-}75 = {}^{+}15$, i.e. 15 km h^{-1} east.

Graphically:

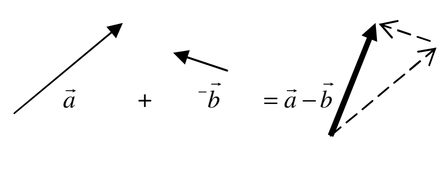


Addition and subtraction of vectors in two and three dimensions

In two and three dimensions, addition is done by placing the head of one arrow to the tail of the other. The order that this is carried out does not affect the resultant.



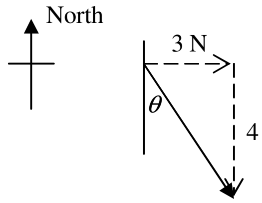
Subtraction is done by changing it to addition first.



Example 4 Two forces, 3 N east and 4 N south act on an object. Find the net force (resultant force) on the object.

The net force is given by the vector addition of the two forces.

Method 1 Draw an accurate scaled diagram, e.g. 1cm : 1 N and measure the length of the resultant vector and its direction.



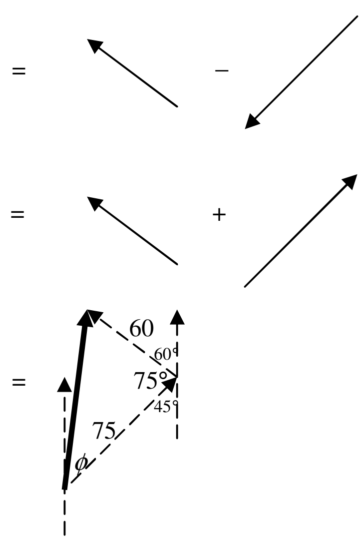
Method 2 Draw a rough sketch and calculate using the trigonometric ratios, the Pythagoras Theorem, the sine or cosine rule.

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 37°$$

$$\therefore \vec{F}_{net} = \sqrt{3^2 + 4^2} = 5 \text{ N S37°E}$$

Example 5 The velocity of a car changes from 75 km h^{-1} SW to 60 km h^{-1} N60°W. What is the change in velocity of the car?

Change in velocity = 60 km h^{-1} N60°W − 75 km h^{-1} SW



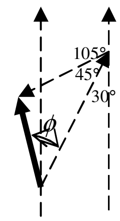
$$|\Delta\vec{v}| = \sqrt{60^2 + 75^2 - 2(60)(75)\cos 75°} \approx 83 \text{ km h}^{-1}.$$

$$\frac{\sin\phi}{60} = \frac{\sin 75°}{83},\ \phi \approx 44°$$

$$\therefore \Delta\vec{v} \approx 83 \text{ km h}^{-1} \text{ N1°E.}$$

Example 6 A car travels 1.2 km N30°E and then 0.7 km S75°W. Find the *displacement* (change in position) of the car from its starting point.

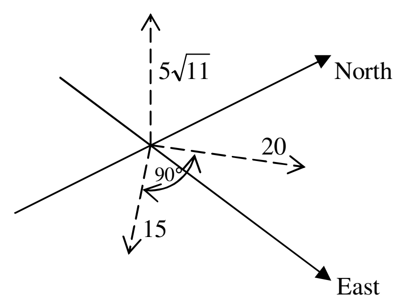
Total displacement $\vec{s}$ from the starting point
= 1.2 km N30°E + 0.7 km S75°W

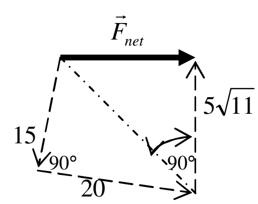


$$|\vec{s}| = \sqrt{1.2^2 + 0.7^2 - 2(1.2)(0.7)\cos 45°} \approx 0.86 \text{ km.}$$

$$\frac{\sin\phi}{0.7} = \frac{\sin 45°}{0.86},\ \phi \approx 35°. \qquad \therefore \vec{s} \approx 0.86 \text{ km N5°W.}$$

Example 7 Three forces, 15 N SE, 20 N NE and $5\sqrt{11}$ N upward act on an object. Find the magnitude of the net force (resultant force) on the object.

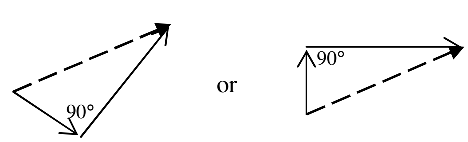




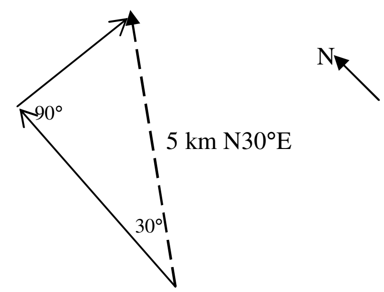
$$F_{net} = \sqrt{15^2 + 20^2 + \left(5\sqrt{11}\right)^2} = 30 \text{ N}$$

Resolving a vector into two perpendicular components

A vector can be decomposed (resolved) into components. In many situations, the most useful way is to resolve a vector into two perpendicular components.



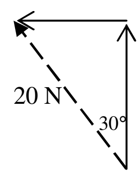
Example 1 A hiker has a displacement of 5 km N30°E. How far to the north and how far to the east is the hiker from her initial position?



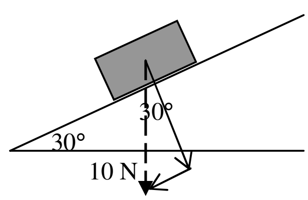
To the north: $5\cos 30° \approx 4.3$ km.
To the east: $5\sin 30° = 2.5$ km.

Example 2 Resolve the 20N force into vertical and horizontal components.

Vertical: $20\cos 30° \approx 17$ N
Horizontal: $20\sin 30° = 10$ N



Example 3 An object slides down a smooth plane inclined at $30°$ to the horizontal. The force of gravity on the object is 10 N. Resolve the force of gravity into two perpendicular components: one parallel to the inclined plane and the other perpendicular to it.



Perpendicular to the plane: $10\cos 30° \approx 8.7$ N.
Parallel to the plane: $10\sin 30° = 5.0$ N.

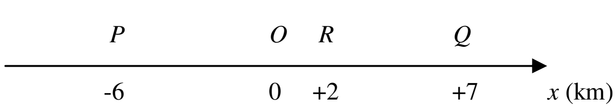
Motion in one dimension

Motion can be described in terms of position, velocity and acceleration. They are vector quantities.

Position

The position of an object is specified in relation to a reference point called the origin. For motion in one dimension, use the number line to indicate positions.

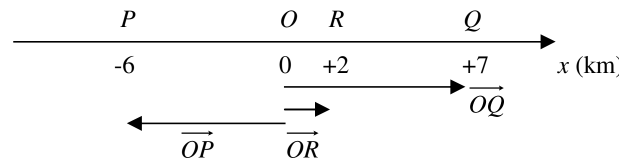
Example 1



Using directed numbers, the position of an object at *P* is -6 km, at *Q* is +7 km, and at *R* is +2 km.

If the number line is running in the east-west direction, the position of an object at *P* is 6 km W, at *Q* is 7 km E, and at *R* is 2 km E.

Using arrows:



Displacement

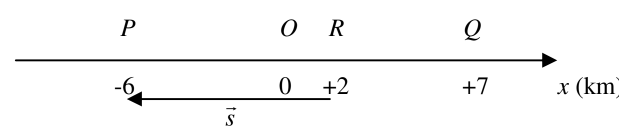
When an object moves its position *x* changes with time *t*. Change in position $\Delta\vec{x}$ is also called displacement $\vec{s}$. It is defined as final position $\vec{x}_f$ – initial position $\vec{x}_i$.

$$\vec{s} = \Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

It is a vector quantity.

Note: Displacement does not depend on the actual path followed by the object. Only the initial and the final positions determine the displacement, e.g. refer to the previous example, if the object moves from *R* through *Q* to *P*, the displacement is $\vec{s} = {}^{-}6 - {}^{+}2 = {}^{-}8$ km or 8 km W.

Graphically, displacement is an arrow of magnitude equal to the straight line distance between initial and final position, and pointing in the direction from initial to final position.



Distance travelled

The distance *d* travelled is the actual length of the path followed by the object. It is a scalar quantity.

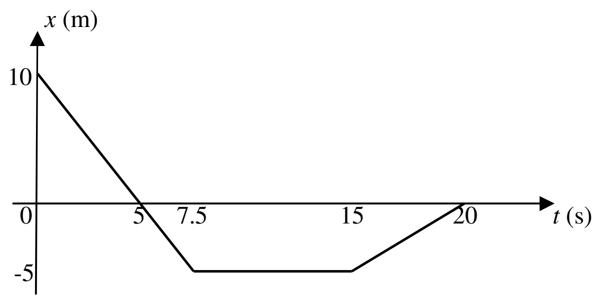
Refer to the previous example, the distance travelled is

$$d = 5 + 13 = 18 \text{ km}.$$

Note: In general, distance travelled does not equal the magnitude of displacement. They are equal only when the object moves in the same direction in the time interval under consideration.

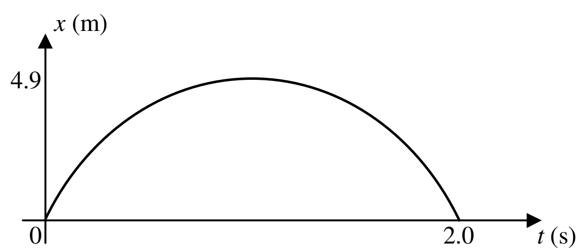
Position-time graph

Example 1 The following graph shows the position of an object at different time. The motion is in the east-west direction. East is chosen to be the positive direction. Describe the motion in terms of positions, directions, displacement and distance travelled.



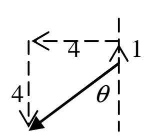
Initially, the object is 10 m east of the origin. It travels 15 m to the west in 7.5 s. Now it is 5 m west of the origin and remains there for another 7.5 s before it turns around and travels 5 m to the east in 5 s, finishing at the origin. Its displacement in the 20 s is -10 m, and the total distance travelled is 20 m.

Example 2 The following graph shows an object in vertical motion. The ground level is chosen as the origin and positions above the ground are considered as positive. Describe the motion in terms of positions, directions, displacement and distance travelled.



The object starts from the ground and moves vertically upwards to a maximum height of 4.9 m in 1.0 s and falls back to the ground in another 1.0 s. The displacement in the 2.0 s is 0 and the total distance travelled is 9.8 m.

Example 3 A person walks 1 km north, 4 km west and then 4 km south.
(a) What is the displacement of the person?
(b) What is the total distance travelled?

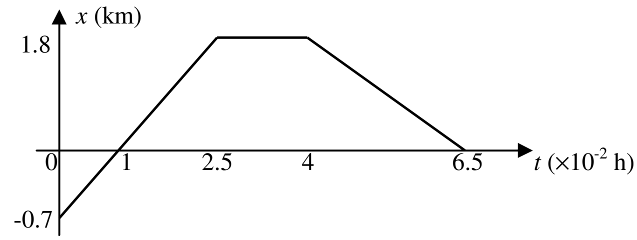


(a) $|\vec{s}| = \sqrt{4^2 + 3^2} = 5$, $\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53°$.

$\therefore \vec{s} = 5$ km S53°W.

(b) $d = 1 + 4 + 4 = 9$ km.

Example 4 A motor-cyclist travelled along a straight road running in the NE direction. The following position-time graph shows the motion of the motor-cyclist. NE is taken as the positive direction. A petrol station is the origin *O*.



(a) What was the initial position of the cyclist?
(b) Where was the cyclist at t = 0.065 h?
(c) For how long was the cyclist at rest?
(d) What was the total distance travelled in the first 0.065 h?
(e) What was the displacement in the first 0.065 h?
(f) In which direction did the cyclist travel at t = 0.050 h?

(a) $\vec{x} = {}^{-}0.7$, i.e. 0.7 km SW of the petrol station.

(b) At the petrol station.

(c) 0.015 h.

(d) $d = 0.7 + 1.8 + 1.8 = 4.3$ km.

(e) $\vec{s} = {}^{+}0.7$ km, i.e. 0.7 km NE.

(f) SW.

Average velocity and average speed

Average velocity and average speed are different quantities and defined differently. Average velocity is a vector quantity while average speed is a scalar.

Definitions:

Average velocity = $\frac{displacement}{timetaken}$, $\vec{v}_{av} = \frac{\vec{s}}{\Delta t}$.

Average speed = $\frac{dis\tan cetravelled}{timetaken}$, $v_{av} = \frac{d}{\Delta t}$.

Example 1 A car travels on a straight road for 30 km at 60 km h^{-1} and then 80 km h^{-1} in the opposite direction for half of an hour. Find its average velocity and average speed.

Forward: 30 km for 0.50 h.
Opposite direction: 40 km for 0.50 h.
Displacement $\vec{s} = {}^{+}30 + {}^{-}40 = {}^{-}10$ km.
Distance travelled $d = 30 + 40 = 70$ km.
Time taken $\Delta t = 0.50 + 0.50 = 1.0$ h.

$\vec{v}_{av} = \frac{{}^{-}10}{1.0} = {}^{-}10$ km h^{-1}. $v_{av} = \frac{70}{1.0} = 70$ km h^{-1}.

Note: Average speed is NOT equal to the magnitude of average velocity in general.

Instantaneous velocity and instantaneous speed

Instantaneous velocity and instantaneous speed are simply called velocity and speed respectively. The speed of a car is given by the speedometer reading and the velocity is given by the speedometer and compass readings. Therefore, speed is equal to the magnitude of velocity.

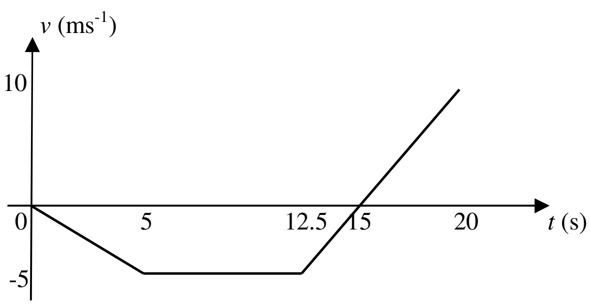
The two quantities can be calculated according to the definitions:

Velocity $\vec{v} \approx \frac{\vec{s}}{\Delta t}$ and speed $v \approx \frac{d}{\Delta t}$ for $\Delta t \to 0$, i.e. very short time interval.

Velocity-time graph

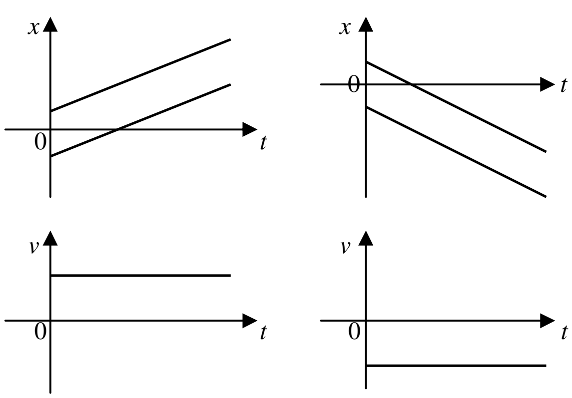
The motion of an object moving in a straight line can be represented by a velocity-time graph.

Example 1 The following graph shows the velocity of an object at different time. The motion is in the east-west direction. East is chosen to be the positive direction. Describe the motion in terms of velocity, speed, direction.



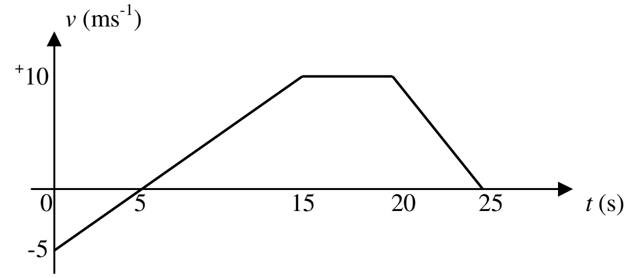
The object starts from rest and travels to the west with its speed increasing uniformly, reaching 5 ms^{-1} for the first 5 s. It maintains this velocity (speed and direction) for 7.5 s before slowing down uniformly to a stop momentarily in the next 2.5 s. It then speeds up uniformly to the east in another 5 s, reaching a speed of 10 ms^{-1}.

Relationship between *v-t* and *x-t* graphs



Gradient of a position-time graph gives velocity, and area 'under' a velocity-time graph gives displacement (not position).

Example 1 From the *v-t* graph below, find the average velocity and the average speed. Take north to be the positive direction.



0-5: $s_1 = \frac{1}{2}\times {}^-5\times 5 = {}^-12.5$ m

5-25: $s_2 = \frac{1}{2}\times (5+20)\times {}^+10 = {}^+125$ m

Displacement $= s_1 + s_2 = {}^+112.5$ m

Average velocity $= \frac{{}^+112.5}{25} = {}^+4.5$, i.e. 4.5 ms^{-1} N.

Distance travelled $= |s_1| + |s_2| = 12.5 + 125 = 137.5$ m

Average speed $= \frac{137.5}{25} = 5.5$ ms^{-1}.

Average acceleration and instantaneous acceleration

Acceleration is the rate of change of velocity. A change in velocity can be the result of a change in speed, direction or both. Acceleration is a vector quantity.

Definitions:

Average acceleration $= \frac{change in velocity}{time taken}$, i.e. $a_{av} = \frac{\Delta v}{\Delta t}$.

Instantaneous acceleration $a \approx \frac{\Delta v}{\Delta t}$ for $\Delta t \to 0$.

The direction of motion is given by the direction of the *velocity vector*. The direction of the *acceleration vector* indicates the direction of the net force acting on the object.

Speeding up or slowing down

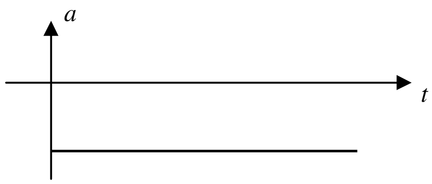
When the velocity and acceleration vectors point in the same direction the object speeds up. When they are opposite in direction, the object slows down.

Speeding up:

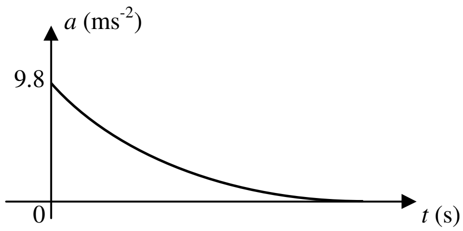
Slowing down:

Acceleration-time graph

Example 1 The following *a-t* graph shows the motion of a ball-bearing projected vertically upwards (taken as the positive direction) under the influence of gravity (assume constant) with negligible air resistance.

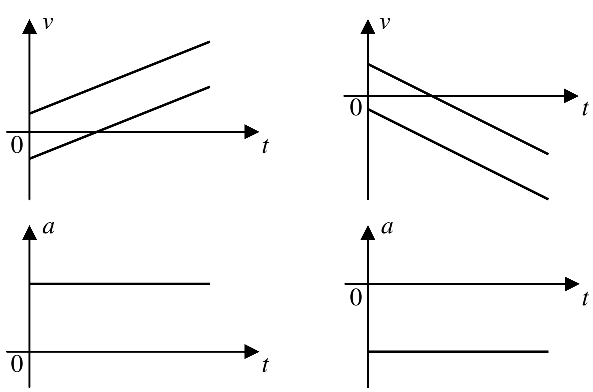


Example 2 The following *a-t* graph shows the motion of a tennis ball dropped from a great height with air resistance. Downward is taken as the positive direction.



When the acceleration reaches zero, the tennis ball falls at constant velocity called its **terminal velocity**.

Relationship between *v-t* and *a-t* graphs



The gradient of a velocity-time graph gives the acceleration, and the area 'under' an acceleration-time graph gives the change in velocity (not velocity).

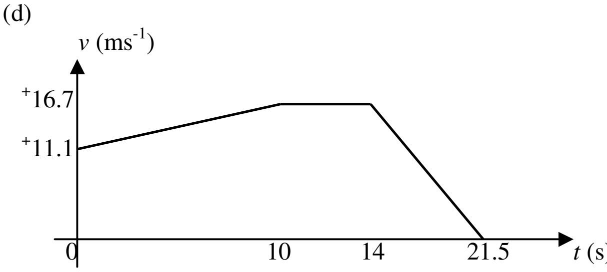
Example 3 Jane travelling east at 40 km h^{-1} increases her speed to 60 km h^{-1} in 10 s.
(a) Calculate her change in velocity in ms^{-1}.
(b) Calculate her average acceleration in ms^{-2}.
Jane travels a further 4.0 s at a constant velocity of 60.0 km h^{-1} east, then slows down to a stop in 7.5 s.
(c) Calculate her average acceleration during the slow down to a stop.
(d) Draw a velocity-time graph for the whole trip, assuming the accelerations are uniform.

(a) Change in velocity= 60 km h^{-1} E – 40 km h^{-1} E = 20 km h^{-1} E ≈ 5.6 ms^{-1} E.

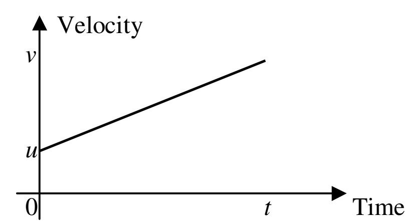
(b) Average acceleration $= \frac{5.6}{10} = 0.56$ ms^{-2} E.

(c) Change in velocity= 0 km h^{-1} E – 60 km h^{-1} E = 60 km h^{-1} W ≈ 16.7 ms^{-1} W.
Average acceleration $= \frac{16.7}{7.5} = 2.2$ ms^{-2} W.

(d)



Motion in a straight line under constant acceleration



u: Initial velocity, i.e. velocity at $t = 0$.
v: Final velocity, i.e. velocity at time *t*.
a: Acceleration (constant).
s: Displacement from the initial position which is the position at $t = 0$.

The gradient of the *v-t* graph gives the acceleration,
i.e. $a = \frac{v-u}{t-0}$, $\therefore v = u + at$ ....(1)
The area under the *v-t* graph gives the displacement,
i.e. $s = \frac{1}{2}(u+v)t$ ....(2)

Eliminating *v* from (1) and (2), $s = ut + \frac{1}{2}at^2$ ....(3)

Eliminating *u* from (1) and (2), $s = vt - \frac{1}{2}at^2$ ....(4)

Eliminating *t* from (1) and (2), $v^2 = u^2 + 2as$ ....(5)

Note: Each equation involves four of the five quantities, *u*, *v*, *a*, *s* and *t*. In these equations *u*, *v*, *a* and *s* are vector quantities, a direction needs to be chosen as positive. These equations are used for motions with constant acceleration. If the acceleration is not constant, they can be used as an approximation by taking the average acceleration as constant acceleration *a*.

Example 1 On a dry road a car with good tyres may be able to slow down at a rate of 4.92 ms^{-1} per second.
(a) How long does it take to come to rest from an initial speed of 24.6 ms^{-1}?
(b) How far does it travel in this time?

Take forward as the positive direction. Since the car slows down, the acceleration vector is opposite to the velocity vector.

(a) $u={}^{+}24.6$ ms^{-1}, $a={}^{-}4.92$ ms^{-2}, $v=0$, find t.
Use $v=u+at$, $0={}^{+}24.6+{}^{-}4.92t$, $t=5.00$ s.

(b) $u={}^{+}24.6$ ms^{-1}, $a={}^{-}4.92$ ms^{-2}, $v=0$, find s.
Use $v^2=u^2+2as$, $0=24.6^2+2({}^{-}4.92)s$, $s={}^{+}61.5$ m.
Distance travelled = 61.5 m.

Example 2 At the instant the traffic light turns green, a car starts with constant acceleration of 2.2 ms^{-1}. At the same instant a truck, travelling at a constant speed of 9.5 ms^{-1}, overtakes and passes the car.
(a) How far beyond the traffic light will the car overtake the truck?
(b) How fast will the car be travelling at that instant?

Take forward as the positive direction. Let T and ^{+}D be the time and displacement when the car overtake the truck

(a) Car: $u=0$, $a={}^{+}2.2$, $t=T$, $s={}^{+}D$.
Use $s=ut+\frac{1}{2}at^2$, ${}^{+}D=\frac{1}{2}({}^{+}2.2)T^2$ ……(1)
Truck: $u={}^{+}9.5$, $a=0$, $t=T$, $s={}^{+}D$.
Use $s=ut+\frac{1}{2}at^2$, ${}^{+}D={}^{+}9.5T$, $\therefore T=\frac{D}{9.5}$ ……(2)
Substitute (2) in (1), $D=1.1\left(\frac{D}{9.5}\right)^2$, $\therefore D=0$ or 82.
Distance travelled = 82 m.

(b) Car: $u=0$, $a={}^{+}2.2$, $s={}^{+}82$, find $|v|$.
Use $v^2=u^2+2as$, $v^2=2\left({}^{+}2.2\right)\left({}^{+}82\right)$, $|v|=19$ ms^{-1}.

Free fall vertical motion

When an object moves under the influence of gravity only, it is in **free fall**. Close to the surface of the earth, acceleration due to gravity can be considered as constant and has an approximate value of 9.8 ms^{-2}. The five equations for constant acceleration can be used in free fall vertical motion. The upward direction is usually taken as the positive direction.

Example 1 Raindrops fall to the ground from a cloud 1700m above. If they were not slowed by air resistance, how fast would the drops be moving just before they hit the ground?

Take downward as the positive direction.
$u=0$, $s={}^{+}1700$, $a={}^{+}9.8$, find v.
Use $v^2=u^2+2as$, $v^2=2\left({}^{+}9.8\right)\left({}^{+}1700\right)$, $|v|\approx 183$ ms^{-1}.

Example 2 A rock is projected vertically upwards from the edge of the top of a tall building. The rock reaches its maximum height 1.60 s after it was launched. Then, after barely missing the edge of the building as it falls downwards, the rock hits the ground 6.00 s after it was launched.
(a) With what upward velocity was the rock projected?
(b) How tall is the building?
(c) What maximum height above the ground was reached?

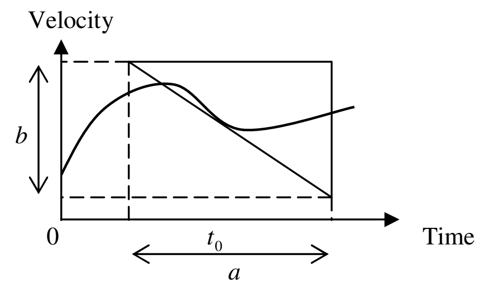
Take upward as the positive direction.
(a) Consider the upward motion:
$t=1.60$, $v=0$, $a={}^{-}9.8$, find u.
Use $v=u+at$, $0=u+{}^{-}9.8\times 1.60$, $u\approx{}^{+}15.7$ ms^{-1}.

(b) Consider the whole trip:
$t=6.00$, $u={}^{+}15.7$, $a={}^{-}9.8$, find s.
Use $s=ut+\frac{1}{2}at^2$, $s=\left({}^{+}15.7\right)(6.00)+\frac{1}{2}\left({}^{-}9.8\right)(6.00)^2$,
$s\approx{}^{-}82.3$. The building is about 82 m tall.

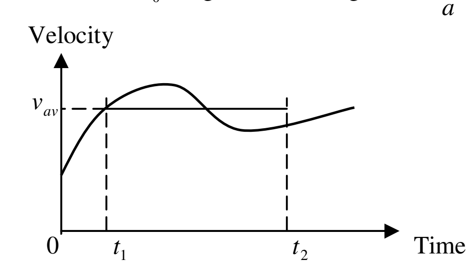
(c) Consider the upward motion:
$t=1.60$, $v=0$, $a={}^{-}9.8$, find s.
Use $s=vt-\frac{1}{2}at^2$, $s=-\frac{1}{2}\left({}^{-}9.8\right)(1.60)^2\approx{}^{+}12.5$ m
Maximum height reached $=12.5+82.3\approx 95$ m.

Non-uniform motion in a straight line

For motion with changing acceleration, graphical analysis is a handy way to study the motion. If v-t graph of motion is known, gradient of tangent to the graph at a particular time gives the acceleration at that time. Area under the graph between t_1 and t_2 gives the displacement in that interval.



Acceleration (at $t=t_0$) = gradient of tangent $=-\frac{b}{a}$



Displacement (between t_1 and t_2) = area under graph $\approx v_{av}(t_2-t_1)$.
Note: v_{av} is the average velocity (by estimation) in the interval between t_1 and t_2.

Effect of a force according to Aristotle, Galileo and Newton (Newton's first law)

To Aristotle, the natural state of a body was to be at rest, and a force was believed necessary to keep a body in uniform motion. The greater the force on the body, the greater is its speed.

Galileo came to a different conclusion some 2000 years later. He imagined an idealised world where there is no **friction**. He claimed that it is just as natural for a body to be in motion with constant velocity as it is to be at rest. He concluded that if no force is applied to a moving body, it will continue to move with constant velocity. A body slows down only if a force is exerted on it. Galileo interpreted friction as a force like pushes and pulls.

Newton's analysis of motion is summarised in his "three laws of motion". In fact, **Newton's first law of motion** is very close to Galileo's conclusions. It states that:

Every body continues in its state of rest or of uniform speed in a straight line unless it is compelled to change that state by forces acting on it.

The tendency of a body to maintain its state of rest or of uniform speed in a straight line is called **inertia**. Newton's first law is often called the **law of inertia**.

Mass and weight of an object

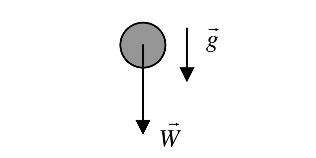
Newton used the term **mass** to stand for quantity of matter in a body. It is measured in kilogram (kg). The more mass a body has, the harder it is to change its velocity.

According to Newton, all bodies are attracted by the earth. This attractive force on a body is called the **force of gravity** on the body or simply the **weight** of the body. It acts at the centre of mass point (approximated as the geometric centre) of the body. At the surface of the earth the force of gravity has an approximate value of 9.8 N for each kilogram mass of the body.

$\therefore \vec{W} = m\vec{g}$, where m is the mass of a body in kg, $\vec{g}$ has the value 9.8 Nkg^{-1} at the surface of the earth, and $\vec{W}$ the weight of the body in N.

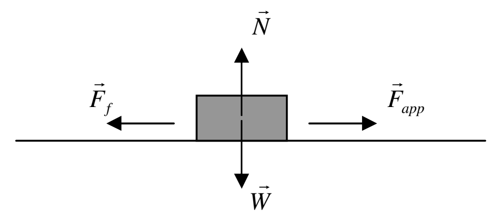
The value of $\vec{g}$ is not constant. It decreases as a body moves away from the earth.

$\vec{W}$ and $\vec{g}$ are both vectors pointing towards the centre of the earth, i.e. downwards.



Friction

When an object is pressed against a surface and an applied force $\vec{F}_{app}$ attempts to slide the object along the surface, the resulting frictional force $\vec{F}_f$ exerted by the surface on the object has the following properties:



Property 1 If the object does not move, the frictional force is equal in magnitude to the applied force, i.e. $F_f = F_{app}$.

Property 2 This friction has a maximum value given by μN , where μ is a numerical value called the **coefficient of friction**, and N is the magnitude of the normal force of the surface on the object, which indicates how hard the object is pressed against the surface.

When F_{app} is greater than the maximum friction, the object begins to slide along the surface. The sliding friction on the object is approximately given by the maximum friction, i.e. $F_f \approx \mu N$.

Change in motion and its cause

A change in motion of a body is caused by the action of a force (net/resultant force if there are more than one force) on the body.

The rate of change in motion (i.e. acceleration $\vec{a}$) of an object of mass m under the action of a net force $\vec{F}_{net}$ is summarised in **Newton's second law**. It states that acceleration is directly proportional to the net force and inversely proportional to the mass of the object, i.e.

$$a \propto F_{net} \text{ and } a \propto \frac{1}{m}.$$

$$\therefore a \propto \frac{F_{net}}{m}, \therefore a = k\frac{F_{net}}{m}.$$

If a is measured in ms^{-1}, F_{net} in N and m in kg, then k = 1, and

$$a = \frac{F_{net}}{m} \text{ or } F_{net} = ma.$$

Furthermore, $\vec{F}_{net}$ and $\vec{a}$ are in the same direction,

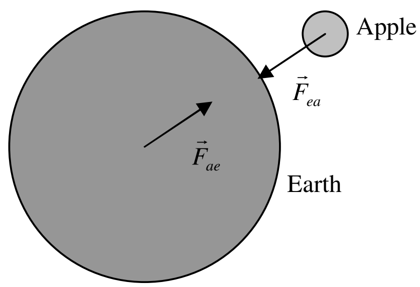
$$\therefore \vec{F}_{net} = m\vec{a}.$$

This is commonly known as Newton's second law of motion.

Newton's Third Law

When two objects A and B interact with each other, if A exerts a force on B, B exerts a force of the same magnitude in the opposite direction on A. This is known as **Newton's third law**. One of the pair of forces is called **action** and the other **reaction**.

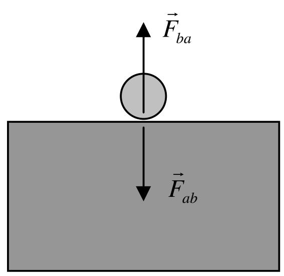
Example 1 An apple falling towards the earth



$\vec{F}_{ea}$: Force of gravity of the earth on the apple.

$\vec{F}_{ae}$: Force of gravity of the apple on the earth.

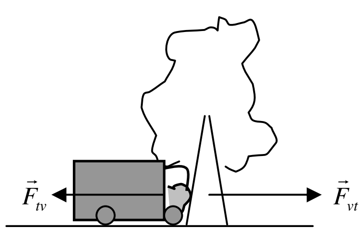
Example 2 An apple resting on a box.



$\vec{F}_{ba}$: Normal force of the box on the apple.

$\vec{F}_{ae}$: Force of the apple on the box.

Example 3 A van colliding with a tree.

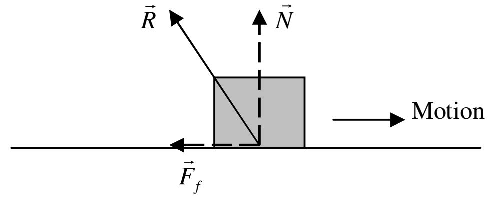


$\vec{F}_{vt}$: Force of the van on the tree.

$\vec{F}_{tv}$: Force of the tree on the van.

Reaction force

When an object is pushed (pulled) along a rough surface, the reaction force $\vec{R}$ on the object consists of two components: one parallel to the surface (force of friction of surface on object) and one perpendicular to the surface (normal force of surface on object).



Note: The diagram above shows only the reaction force $\vec{R}$. There are other forces (not shown) on the object, e.g. air resistance, force of gravity.

Application of Newton's laws

Example 1 A student pushes a loaded sled of mass 240 kg on the frictionless surface of a frozen lake. She exerts a constant horizontal force of 130 N. If the sled starts from rest, what is its velocity after sliding 2.3m?

Force of gravity and normal force are equal and opposite in direction. They add to zero. $\therefore$net force equals pushing force.

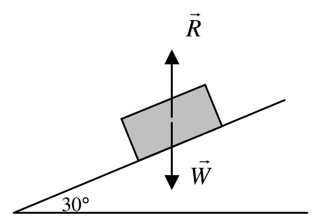
$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{^{+}130}{240} \approx {}^{+}0.542$, $u = 0$, $s = {}^{+}2.3$, find v.

Use $v^2 = u^2 + 2as$, $v \approx {}^{+}1.6$, i.e. 1.6 ms^{-1} forward.

Example 2 A 2-kg object is at rest on an plane inclined at 30° to the horizontal.

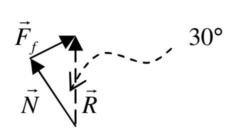
(a) Determine the magnitude and direction of the reaction force of the plane on the object.

(b) What are the magnitudes of the normal force and friction?



(a) Reaction force $\vec{R}$: $R = W = mg = 2 \times 9.8 \approx 20$ N, vertically upward.

(b) $\vec{R}$ on the object consists of two components: one parallel to the surface (force of friction of surface on object) and one perpendicular to the surface (normal force of surface on object).



$F_f \approx 20\sin 30° = 10$ N, $N \approx 20\cos 30° = 17$ N.

Example 3 A crate of mass 360 kg rests on the bed of a truck moving at a speed of 120 km h^{-1}. The driver applies the brakes and slows to a speed of 62 km h^{-1} in 17 s. What average force acts on the crate during this time, assuming that the crate does not slide on the truck bed?

120 km h^{-1} $=\dfrac{120}{3.6}=33.3$ ms^{-1}, 62 km h^{-1} $=17.2$ ms^{-1}.

$u={}^{+}33.3$, $v={}^{+}17.2$, $t=17$, find a.

Use $v=u+at$, $a=\dfrac{v-u}{t}={}^{-}0.95$ ms^{-2}.

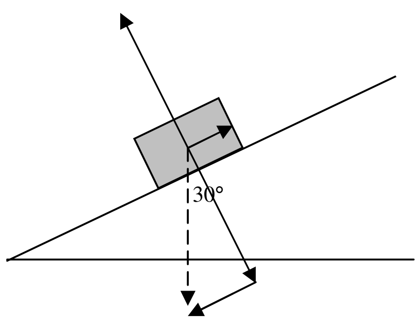
Average force $=ma=360\times{}^{-}0.95\approx{}^{-}341$ N, i.e. 341 N opposite to the direction of motion.

Example 4 A 2-kg object slides down a plane inclined at 30° to the horizontal. The coefficient of friction between the object and the plane is $\mu=0.2$.
(a) Calculate the friction between the object and the plane.
(b) Calculate the magnitude of the reaction force of the plane on the object.
(c) Using components parallel and perpendicular to the plane, determine the net force on the object.
(d) Determine the acceleration of the object.

(a) $N=mg\cos 30°\approx 17$, $F_f=\mu N\approx 3.4$ N.

(b) $R=\sqrt{3.4^2+17^2}=17.3$ N.

(c)



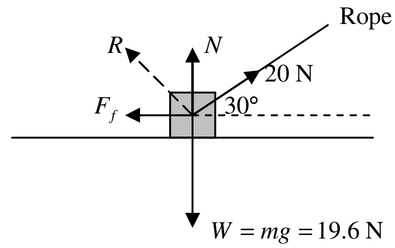
Component perpendicular to the plane = 0.
Component parallel to the plane $={}^{+}mg\sin 30°+{}^{-}F_f$

$\approx{}^{+}10+{}^{-}3.4={}^{+}6.6$ N.

$\therefore \vec{F}_{net}\approx 6.6$ N down the plane.

(d) $\vec{a}=\dfrac{\vec{F}_{net}}{m}\approx\dfrac{{}^{+}6.6}{2}={}^{+}3.3$ ms^{-2}, i.e. 3.3 ms^{-2} down the plane.

Example 5 A 2-kg object is pulled along a horizontal floor with a rope making an upward angle of 30° to the floor. The tension in the rope is 20 N and $\mu=0.3$.
(a) Determine the normal force of the floor on the object.
(b) Calculate the friction force against the object's motion.
(c) Determine the reaction force R of the floor on the object.
(d) Calculate the net force on the object.



Note: Reaction force $\vec{R}$ is resolved into $\vec{N}$ and $\vec{F}_f$ in the diagram.

(a) Apply Newton's first law to the vertical component:
${}^{+}N+{}^{+}20\sin 30+{}^{-}19.6=0$, $N=9.6$ N.

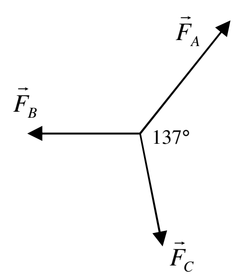
(b) $F_f=\mu N=0.3\times 9.6\approx 2.9$ N.

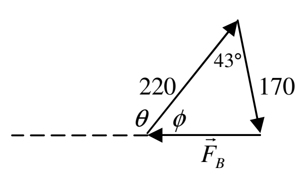
(c) $R=\sqrt{2.9^2+9.6^2}\approx 10$ N.

(d) Horizontal component: $\vec{F}_{net}={}^{+}20\cos 30°+{}^{-}2.9\approx{}^{+}14$ N.

Example 6 An object on a frictionless horizontal surface is pulled by three horizontal forces: $\vec{F}_A=220$ N, $\vec{F}_C=170$ N and $\vec{F}_B$ is unknown. $\vec{F}_A$ and $\vec{F}_C$ make an angle of 137°, and the object remains stationary. Find the magnitude and direction of $\vec{F}_B$.

Newton's first law: $\vec{F}_A+\vec{F}_B+\vec{F}_C=\vec{0}$.



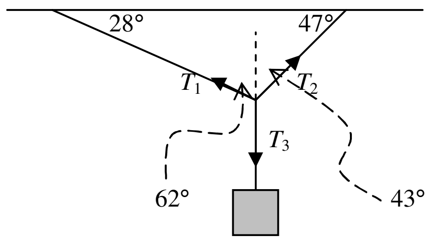


$F_B=\sqrt{220^2+170^2-2(220)(170)\cos 43°}\approx 150$ N.

$\dfrac{\sin\phi}{170}=\dfrac{\sin 43°}{150}$, $\phi\approx 51°$, $\therefore \theta\approx 129°$.

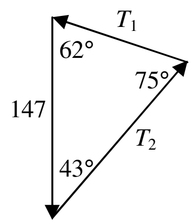
$\vec{F}_B$ is 150 N and makes an angle of 129° with $\vec{F}_A$.

Example 7 A block of mass 15 kg hangs from three cords. What are the tensions in the cords?



$T_3 = W = mg = 15 \times 9.8 = 147$ N.

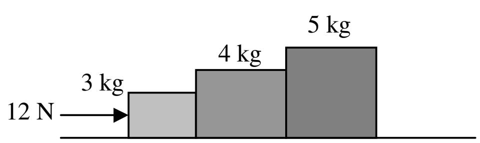
Apply Newton's first law: $\vec{T}_1 + \vec{T}_2 + \vec{T}_3 = \vec{0}$.



$\frac{T_1}{\sin 43°} = \frac{147}{\sin 75°}$, $\frac{T_2}{\sin 62°} = \frac{147}{\sin 75°}$.

$\therefore T_1 \approx 104$ N, $T_2 \approx 134$ N.

Example 8



Three boxes, 3 kg, 4 kg and 5 kg, are pushed from the left with a force of 12 N on a frictionless surface.

(a) Determine the common acceleration of the boxes.

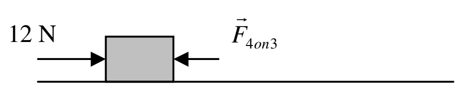
(b) Find the force $\vec{F}_{3on4}$ that the 3-kg box exerts on the 4-kg box.

(c) Find the force $\vec{F}_{4on5}$ that the 4-kg box exerts on the 5-kg box.

(a) Consider the three boxes as a single object.

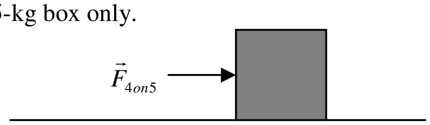
$a = \frac{F_{net}}{m} = \frac{12}{3+4+5} = 1.0$ ms^{-2}.

(b) Consider the 3-kg box only.



$^{+}12 + \vec{F}_{4on3} = 3 \times {}^{+}1.0$, $\vec{F}_{4on3} = {}^{-}9.0$ N, $\therefore \vec{F}_{3on4} = 9.0$ N.

(c) Consider the 5-kg box only.



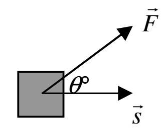
$\vec{F}_{4on5} = 5 \times {}^{+}1.0 = {}^{+}5.0$ N.

Work done by a constant force

Work done W by a constant force $\vec{F}$ on an object over a displacement $\vec{s}$ is defined as $W = Fs$ if $\vec{F}$ and $\vec{s}$ are in the same direction. W in this situation is a positive amount of work and the object gains energy.

If $\vec{F}$ and $\vec{s}$ are in opposite directions. W is a negative amount of work, $W = -Fs$ and the object loses energy.

If $\vec{F}$ and $\vec{s}$ are at an angle $\theta°$, the work done is given by $W = Fs\cos\theta°$.



$W = Fs\cos\theta°$ can be interpreted in two ways:

(1) $W = Fs\cos\theta° = (F\cos\theta°)s$, i.e. displacement $s \times$ the component of $\vec{F}$ in the direction of $\vec{s}$.

(2) $W = Fs\cos\theta° = F(s\cos\theta°)$, i.e. force $F \times$ distance moved in the direction of $\vec{F}$.

Work and energy are scalar quantities and they are measured in joules (J).

Example 1 A crate is pulled along a rough level surface with a rope at an angle of 25° above the horizontal. The tension in the rope is 100 N and friction force against motion is 80 N.

(a) Find the work done by the pulling force when the crate moves a distance of 1.2 m.

(b) Find the work done by the friction force over the 1.2 m.

(c) Find the work done by the normal force on the crate.

(d) Find the work done by the net force over the 1.2 m.

(a) $W = Fs\cos\theta° = 100 \times 1.2 \times \cos 25° \approx 110$ J (108.8 J).

(b) $W = -Fs = -80 \times 1.2 = -96$ J. The negative sign indicates that energy is taken out of the system.

(c) $W = Fs\cos\theta° = N \times 1.2 \times \cos 90° = 0$, where N is the normal force on the crate.

(d) $\vec{F}_{net} = {}^{+}100\cos 25° + {}^{-}80 = {}^{+}10.63$ N.

$W = F_{net}s = 10.63 \times 1.2 \approx 13$ J.

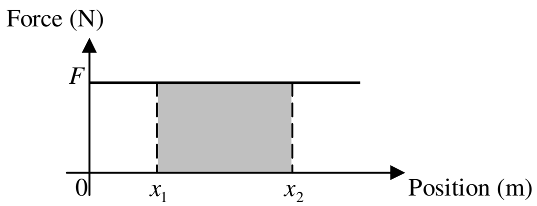
Alternatively, $W = 108.8 - 96 \approx 13$ J.

Example 2 How much work is done by the force of gravity of the earth on the moon in a month?

Force of gravity does no work on the moon because the force of gravity on the moon and the motion (velocity) of the moon are always perpendicular in direction.

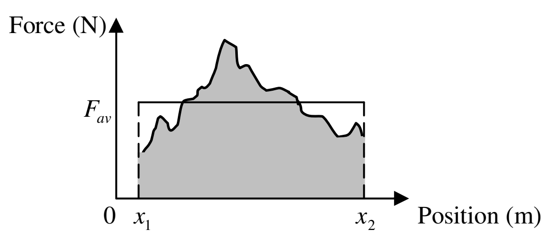
Area under force-position graph

When force $\vec{F}$ on an object is constant and in the same direction as $\vec{s}$, the rectangular area under the force-position graph from x_1 to x_2 represents the work done on the object by $\vec{F}$.



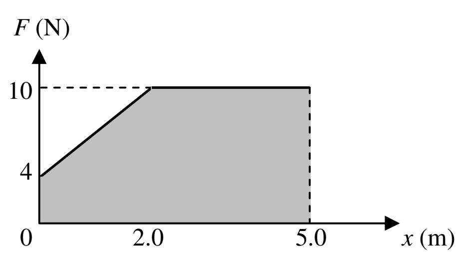
$W = area = F(x_2 - x_1) = Fs$.

When force $\vec{F}$ on an object changes with its position, area under the force-position graph still represents work done by $\vec{F}$ if $\vec{F}$ and $\vec{s}$ are in the same direction. Estimate the area if it cannot be determined by simple calculation.



The estimated area $\approx F_{av}(x_2 - x_1) = F_{av} s$.

Example 1 A 8.0-kg block moves in a straight line on a horizontal frictionless surface under the action of a force that varies with position (see graph below). How much work is done by the force as the block moves from the origin to $x = 5.0$ m?



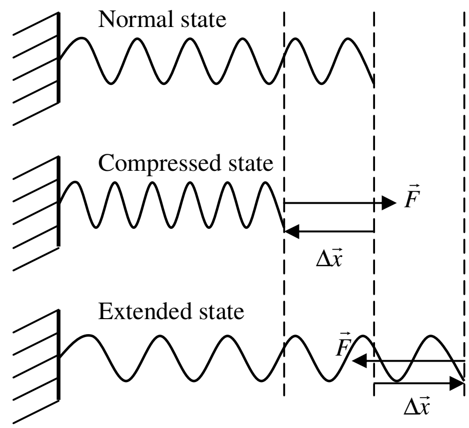
$W = area = 10 \times 5.0 - \frac{1}{2} \times 6 \times 2.0 = 44$ J.

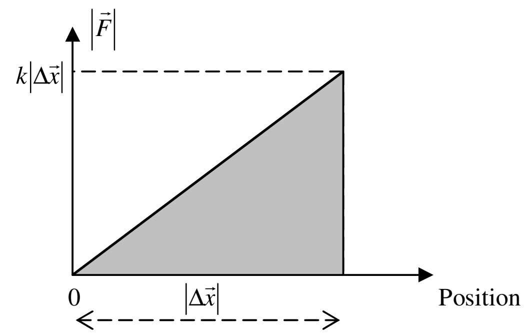
Hooke's law, an example of variable force

For an ideal spring the force exerted by the spring is directly proportional to its compression (or extension):

$$\vec{F} = -k\Delta\vec{x}$$

Note: In $\vec{F} = -k\Delta\vec{x}$, $\vec{F}$ is the force exerted by the spring, **not** the force in compressing or extending the spring. The minus sign indicates that $\vec{F}$ and $\Delta\vec{x}$ are in opposite directions. The proportionality constant k (Nm^{-1}) is called **spring constant**.





The area of the shaded region gives the work done in compressing or extending the spring by $|\Delta\vec{x}|$.

$$W = \frac{1}{2}k|\Delta\vec{x}|^2$$

Example 1 A spring is extended by 2.0 cm when a 5.0-kg load is suspended from it.
(a) Find the force required to stretch it by 5.0 cm.
(b) Determine the amount of work required to stretch it by 5.0 cm.
(c) Find the extra work required to stretch it by another 1.0 cm.

(a) $|\vec{F}| = k|\Delta\vec{x}|$, $5.0 \times 9.8 = k \times 0.020$, $k = 2450\,Nm^{-1}$.

$|\vec{F}| = 2450 \times 0.050 \approx 123$ N.

(b) $W = \frac{1}{2}k|\Delta\vec{x}|^2 = \frac{1}{2} \times 2450 \times 0.050^2 \approx 3.06$ J.

(c) $W = \frac{1}{2} \times 2450 \times 0.060^2 = 4.41$ J.

Extra work $\approx 4.41 - 3.06 = 1.35$ J.

Different kinds of energy

Elastic potential energy: Energy is stored in a spring when it is compressed or extended.
This energy is called **elastic potential energy**. For an ideal spring it is given by

$$E_{ep} = \frac{1}{2}k\left|\Delta\vec{x}\right|^2 .$$

It is equal to the work done in compressing or extending the spring by $\left|\Delta\vec{x}\right|$.

Kinetic energy: It is associated with the state of motion of an object. A moving object possesses kinetic energy E_k (J) and the amount depends on the mass m (kg) and the speed v (ms^{-1}) of the object. By definition,

$$E_k = \frac{1}{2}mv^2 .$$

The definition suggests that $E_k \propto m$ and $E_k \propto v^2$.

When the speed is the same, if the mass of an object is twice that of the other object, then it has twice the amount of kinetic energy than the other.

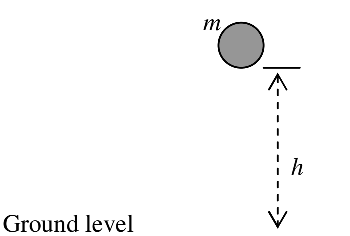
An object with twice the speed has four times the amount of kinetic energy.

Example 1 The speed of a car increases from 50 km h^{-1} to 100 km h^{-1}. Find the value of the ratio
kinetic energy at 100 km h^{-1} : kinetic energy at 50 km h^{-1}.

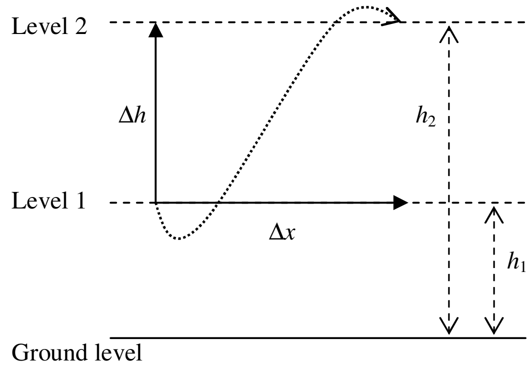
Since $E_k \propto v^2$, $\therefore \frac{E_{k,100}}{E_{k,50}} = \frac{100^2}{50^2} = 4$.

Gravitational potential energy: It is associated with the state of separation between objects that attract each other due to the gravitational force, e.g. between an apple and the earth.

The gravitational potential energy at ground level is arbitrary chosen as zero. By definition, the gravitational potential energy E_{gp} (J) of an object of mass m (kg) at a height h (m) above the ground is $E_{gp} = mgh$.



It is the vertical displacement that determines the change in gravitational potential energy. Horizontal displacement does not change the gravitational potential energy of an object.



When the object moves from level 1 to level 2, its gravitational potential energy increases, and

$$\Delta E_{gp} = mgh_2 - mgh_1 = mg(h_2 - h_1) = mg\Delta h .$$

When it moves from level 2 to level 1 its potential energy decreases by the same amount.

Example 1 A 70-kg person is carried by an escalator to the upper floor 4.5 m above.
(a) What is the increase in gravitational potential energy of the person?
(b) The person later takes a lift down to the lower floor. What is the decrease in her gravitational potential energy?

(a) $\Delta E_{gp} = mg\Delta h = 70\times 9.8\times 4.5 \approx 3.1\times 10^3$ J.

(b) Same amount.

The law of conservation of energy

Energy changes from one form to another and can be transferred from one object to another during interaction. Work is done in the transformation or transfer of energy, e.g. force of gravity does work on an object when the object falls, changing its gravitational potential energy to kinetic energy.

$$E_{gp} \xrightarrow{\text{Work}} E_k$$

The total amount of energy ($E_{gp} + E_k$) at any time during the fall is constant, i.e.

$$mgh_a + \frac{1}{2}mv_a{}^2 = mgh_b + \frac{1}{2}mv_b{}^2 \text{ or } \Delta E_k + \Delta E_{gp} = 0 .$$

This is known as the **law of conservation of energy**.

In the case of a moving object compressing a spring or extending a rubber cord, the law of conservation of energy can be expressed in terms of any two or all three of E_k, E_{ep} and E_{gp}.

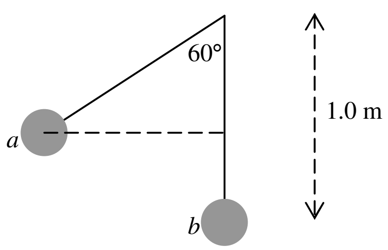
If only E_k and E_{ep} are involved in the situation,

$$\frac{1}{2}mv_a^2+\frac{1}{2}k(\Delta x_a)^2=\frac{1}{2}mv_b^2+\frac{1}{2}k(\Delta x_b)^2 \quad \text{or}$$
$$\Delta E_{gp}+\Delta E_{ep}=0.$$

If all three types of energy are involved,

$$mgh_a+\frac{1}{2}mv_a^2+\frac{1}{2}k(\Delta x_a)^2=mgh_b+\frac{1}{2}mv_b^2+\frac{1}{2}k(\Delta x_b)^2 \quad \text{or}$$
$$\Delta E_k+\Delta E_{gp}+\Delta E_{ep}=0.$$

Example 1 A simple pendulum is released from position *a* to *b* (lowest point). Find the speed of the pendulum bob at *b*.

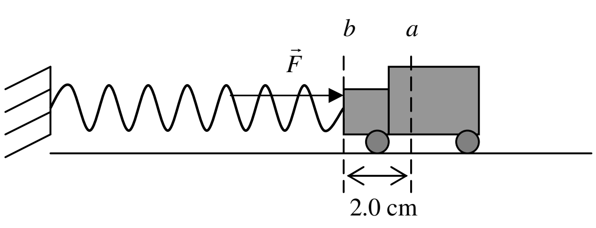


From *a* to *b* the distance fallen $=1.0-1.0\cos 60°=0.5$ m.

$$mgh_a+\frac{1}{2}mv_a^2=mgh_b+\frac{1}{2}mv_b^2,$$

$$m(9.8)(0.5)=\frac{1}{2}mv_b^2,\ v_b\approx 3.1\,\text{ms}^{-1}.$$

Example 2 A toy car (0.25 kg) moving at 1.5 ms^{-1} hits a spring, causing a maximum compression of 2.0 cm to the spring. Find the maximum force exerted by the spring on the toy car.

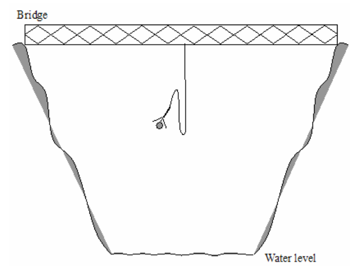


$$\frac{1}{2}mv_a^2+\frac{1}{2}k(\Delta x_a)^2=\frac{1}{2}mv_b^2+\frac{1}{2}k(\Delta x_b)^2,$$

$$\frac{1}{2}(0.25)(1.5^2)=\frac{1}{2}k(^{-}0.020)^2,\ \therefore k\approx 1.4\times 10^3\ \text{Nm}^{-1}.$$

$$\vec{F}=-k\Delta\vec{x},\ \vec{F}=-(1.4\times 10^3)(^{-}0.020)\approx {}^{+}28\ \text{N}.$$

Example 3 A 72.0-kg person attempts a bungee jump. The bungee cord is 35 m long and it is elastic (i.e. it follows Hooke's law) with $k=250\,\text{Nm}^{-1}$. Air resistance is to be ignored. Consider the person as a point mass starting **from rest**, and just reaching the water.



(a) How high is the bridge above the water?
(b) Determine the speed of the person at 45 m below the bridge.

(a) Let *x* be the vertical distance between the bridge and the water. The person has gravitational potential energy at the bridge, and elastic potential energy at the water level.

$mgh_a+\frac{1}{2}k(\Delta x_a)^2=mgh_b+\frac{1}{2}k(\Delta x_b)^2$, where *a* stands for at the bridge, and *b* at the water level.

$$72.0(9.8)x=\frac{1}{2}\times 250(x-35)^2,$$

$$5.6448x=(x-35)^2,\ x\approx 52\text{ m}.$$

(b) $mgh_a+\frac{1}{2}mv_a^2+\frac{1}{2}k(\Delta x_a)^2=mgh_b+\frac{1}{2}mv_b^2+\frac{1}{2}k(\Delta x_b)^2$, where *a* stands for at the bridge, and *b* at 45 m below the bridge.

$$72.0(9.8)(52)=72.0(9.8)(52-45)+\frac{1}{2}(72.0)v_b^2+\frac{1}{2}(250)(45-35)^2,$$

$$v_b\approx 23\text{ ms}^{-1}.$$

Power

Power is a scalar quantity that measures the rate at which work is done by a force, or energy is transferred or transformed.

Average power is defined as $P_{av}=\frac{W}{\Delta t}$, or $P_{av}=\frac{\Delta E}{\Delta t}$, where W (J) is the amount of work done, ΔE (J) amount of energy transferred and Δt (s) the time taken.

Power is measured in joules per second or watts (J s^{-1}, or W).

Example 1 A load of bricks (420 kg) is to be lifted by a winch to a height of 20 m in 1.0 min. What must be the minimum power of the winch motor?

$$\Delta E_{gp}=420\times 9.8\times 20=82320\text{ J}$$

$$P=\frac{\Delta E}{\Delta t}=\frac{82320}{60}\approx 1.4\times 10^3\text{ W}$$

The actual power must be greater than this because friction and other retarding forces work against the lift. The winch is not 100% efficient. A fraction of the total amount of work done by the motor is used to lift the bricks, and the rest changes to heat and sound.

Efficiency

$$Efficiency = \frac{useful, amount, of, work, done}{total, amount, of, work, done} \times 100\% ,$$

or $$Efficiency = \frac{useful, energy, transferred}{total, energy, \exp ended} \times 100\% ,$$

or $$Efficiency = \frac{useful, power}{total, power} \times 100\% .$$

Example 1 A motor produces 9000 J of heat while performing 2700 J of useful work. What is the efficiency of the motor?

Total energy output of motor $= 2700 + 9000 = 11700$ J.

Efficiency $= \frac{2700}{11700} \times 100\% \approx 23\%$.

Example 2 A 38-percent-efficient power plant puts out 700 MW of electrical power.
(a) What is the rate of energy consumption of the power plant?
(b) How much heat is released into the atmosphere in an hour?

(a) $Efficiency = \frac{useful, power}{total, power} \times 100\%$,

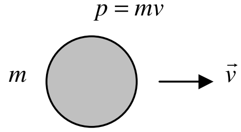
$38\% = \frac{700}{P_{total}} \times 100\%$, $P_{total} \approx 1.8 \times 10^3$ MW.

(b) In an hour total energy expended
$E_{total} = P_{total} \times 1 \approx 1.8 \times 10^3$ MWh.
Amount of heat $= 62\% \times E_{total} \approx 1.1 \times 10^3$ MWh.

Impulse and momentum

Momentum $\vec{p}$ of an object is a vector quantity which is defined as the product of mass and velocity of the object.

$$\vec{p} = m\vec{v}$$



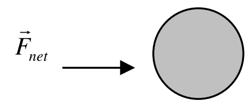
Momentum is measured in kg ms^{-1}.

$\vec{p}$ is in the same direction as $\vec{v}$.

Momentum of an object changes when there is a net force $\vec{F}_{net}$ on the object for a period of time Δt .
The net force causes the object to accelerate (i.e. to change its velocity). Hence there is a change in momentum.
An **impulse** $\vec{I}$ is given to the object by the net force.

Impulse is a vector quantity which is defined as the product of net force and time.

$$\vec{I} = \vec{F}_{net} \Delta t$$



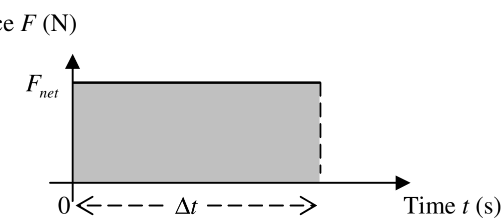
Impulse is measured in Ns.

$\vec{I}$ is in the same direction as $\vec{F}_{net}$.

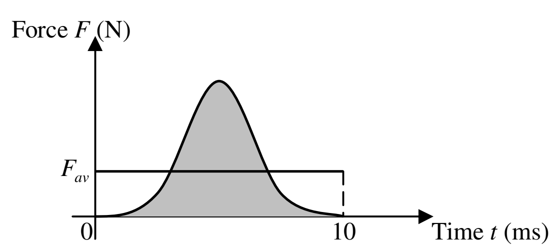
Impulse changes the momentum of an object, and $\vec{I} = \Delta\vec{p}$, where $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v} - m\vec{u}$ is the change in momentum.

Area under force-time graph

The area under a force-time graph represents impulse exerted on an object by a net force. If the force is constant, the area is rectangular and therefore equals $F_{net}\Delta t$.



Very often the force exerted on an object is not constant, e.g. the club head hitting the golf ball in the following example 2.



The impulse exerted on the golf ball is given by the area under the F-t graph. This area can be approximated by the area under the horizontal line F_{av} (average force).

Example 1 A 50-kg crate is initially at rest on the floor. A horizontal net force of 20 N acts on it for 2.0 s.
(a) What is the initial momentum of the crate?
(b) What is its momentum at the end of the first two seconds?
(c) What is its velocity at the end of the first two seconds?

(a) The crate is initially at rest, $\therefore \vec{p}_i = \vec{0}$.

(b) $\vec{I} = \Delta\vec{p}$, $\vec{F}_{net}\Delta t = \vec{p}_f - \vec{p}_i$, ${}^{+}20\times 2.0 = \vec{p}_f - 0$,
$\vec{p}_f = {}^{+}40$ kg ms^{-1}.

(c) $\vec{p}_f = m\vec{v}$, ${}^{+}40 = 50\vec{v}$, $\vec{v} = {}^{+}0.80$ ms^{-1}.

Example 2 A club head hits a 50-g golf ball with an average force of 500 N. The impact time is 10 ms. Determine
(a) Determine the impulse exerted on the ball.
(b) What is the resulting change in momentum of the ball?
(c) Find the velocity of the ball as it leaves the club head.

(a) $\vec{I} = \vec{F}_{net,av}\Delta t = {}^{+}500\times\left(10\times 10^{-3}\right) = {}^{+}5.0$ Ns.

(b) $\Delta\vec{p} = \vec{I} = {}^{+}5.0$ kg ms^{-1}.

(c) $\Delta\vec{p} = m\vec{v} - m\vec{u}$, ${}^{+}5.0 = 0.050\vec{v} - \vec{0}$, $\vec{v} = {}^{+}100$ ms^{-1}.

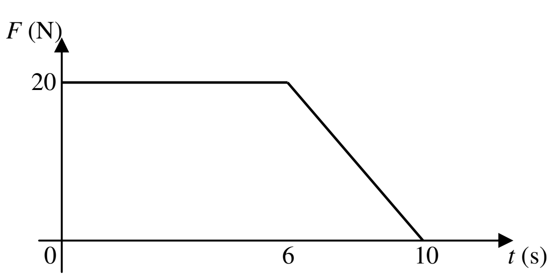
Example 3 A 140-g baseball, in horizontal flight with a speed of 39 ms^{-1}, is struck with a bat. After leaving the bat, the ball travels in the opposite direction with the same speed.
(a) What impulse acts on the ball while it is in contact with the bat?
(b) The impact time for the ball-bat collision is 1.2 ms. What average force acts on the ball?
(c) What is the average acceleration of the ball?

(a) $\vec{I} = \Delta\vec{p} = m\vec{v} - m\vec{u} = m(\vec{v} - \vec{u}) = 0.14\left({}^{-}39 - {}^{+}39\right) \approx {}^{-}11$ Ns.

(b) $\vec{I} = \vec{F}_{net,av}\Delta t$, ${}^{-}11 \approx \vec{F}_{net,av}\times\left(1.2\times 10^{-3}\right)$,
$\vec{F}_{net,av} \approx {}^{-}9.2\times 10^{3}$ N.

(c) $\vec{a}_{av} = \dfrac{\vec{F}_{net,av}}{m} \approx \dfrac{{}^{-}9.2\times 10^{3}}{0.14} \approx {}^{-}6.5\times 10^{4}$ ms^{-2}.

Example 1 An object of mass 3.2 kg and initially at rest is acted on by a net eastward force which varies with time as shown in the graph below.



For the 10-s interval find
(a) the impulse exerted on the object;
(b) the change in momentum of the object;
(c) the change in velocity of the object;
(d) the final velocity of the object.

(a) $\vec{I} = area = \frac{1}{2}(6+10)\times{}^{+}20 = {}^{+}160$ Ns.

(b) $\Delta\vec{p} = \vec{I} = {}^{+}160$ kg ms^{-1}.

(c) $\Delta\vec{p} = m\vec{v} - m\vec{u} = m\Delta\vec{v}$, $\therefore \Delta\vec{v} = \dfrac{\Delta\vec{p}}{m} = \dfrac{{}^{+}160}{3.2} = {}^{+}50$ ms^{-1}.

(d) $\Delta\vec{v} = \vec{v} - \vec{u}$, ${}^{+}50 = \vec{v} - 0$, $\vec{v} = {}^{+}50$ ms^{-1}.

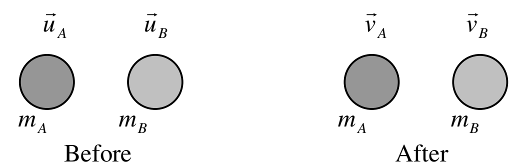
Conservation of momentum

The law of conservation of momentum states that for an isolated system of particles, total momentum (vector sum of momenta of all particles in the system) remains constant.

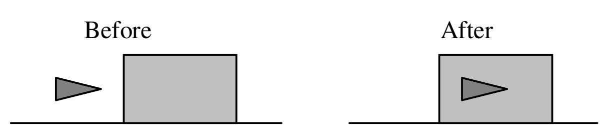
For example, in a collision of two objects A and B, total momentum before collision, during collision and after collision remains the same. This can be expressed as

$$m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B \,.$$

where $\vec{u}$ and $\vec{v}$ are the velocities before and after collision respectively.

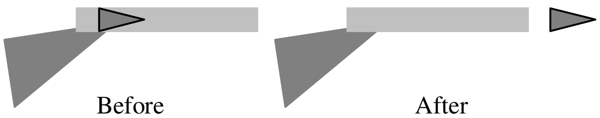


Example 1 A bullet of mass 3.8 grams is fired horizontally with a speed 1100 ms^{-1} into a 12-kg block of wood that is initially at rest on a horizontal table. If the block is free to slide without friction across the table, what speed will it have acquired after it has absorbed the bullet?



Let $\vec{v}$ be the common velocity of the block with the bullet embedded in it.
$m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B$,
$0.0038\times{}^{+}1100 + 12\times 0 = 0.0038\vec{v} + 12\vec{v}$,
$\vec{v} \approx {}^{+}0.35$. Speed ≈ 0.35 ms^{-1}.

Example 2 Calculate the recoil velocity of a 4.0-kg rifle that shoots a 50-g bullet at a speed of 280 ms^{-1}.

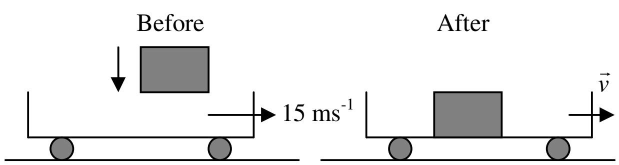


$\vec{0} = 4.0\vec{v} + 0.050 \times {}^{+}280$, $\vec{v} \approx {}^{-}0.29$.
Recoil velocity ≈ 0.29 ms^{-1} backwards.

Example 3 A 10-g bullet travelling at 400 ms^{-1} penetrates a 2.0-kg block of wood and emerges going at 350 ms^{-1}. If the block is stationary when hit, how fast does it move after the bullet emerges?

$0.010 \times {}^{+}400 + 2.0 \times 0 = 0.010 \times {}^{+}350 + 2.0\vec{v}$, $\vec{v} = {}^{+}0.25$.
Speed = 0.25 ms^{-1}.

Example 4 A 10000-kg railroad car travels on a level frictionless track at a constant speed of 15 ms^{-1}. An additional 6000-kg load is dropped onto the car. What will its speed be now?



The vertical momentum of the load is transferred to the earth, and it does not affect the total horizontal momentum.

Horizontally: $6000 \times 0 + 10000 \times {}^{+}15 = (6000 + 10000)\vec{v}$,
$\vec{v} \approx {}^{+}9.4$. Speed ≈ 9.4 ms^{-1}.

