Physics worksheet solutions – Projectile motion **(g = 9.8 ms^{-2}, no air resistance)**

Q1 A golf ball is projected vertically with a speed of 15 ms^{-1}. Calculate the maximum height reached by the ball.

Vertically: $u = +15$, $a = -9.8$, $v = 0$, *use* $v^2 = u^2 + 2as$ *to find s.* .: $0 = 225 - 19.6s$, $s \approx +11.5$

Maximum height ≈ 11.5 m

Q2 A tennis ball is projected from a height of 1.5 m parallel to a horizontal ground. The speed of projection is 25 ms^{-1}. Calculate the maximum horizontal distance reached by the ball the first time it hits the ground.

Vertically: $u = 0$, $a = -9.8$, $s = -1.5$, *use* $s = ut + \frac{1}{2}at^2$ *to find t.* .: $-1.5 = \frac{1}{2}(-9.8)t^2$, $t \approx 0.5533$ s

Horizontally: $u = +25$, $t \approx 0.5533$, $s = ut = +25 \times 0.5533 \approx +13.8$ m. *Maximum distance* ≈ 13.8 m

Q3 It takes 3.0 s for a soccer ball kicked at ground level to hit the ground. Calculate the maximum height reached by the ball.

Vertically: $a = -9.8$, $v = 0$, $t = 1.5$, *use* $s = vt - \frac{1}{2}at^2$ *to find s.*

.: $s = -\frac{1}{2}(-9.8) \times 1.5^2 \approx +11.0$ m

Maximum height ≈ 11.0 m

Q4 Calculate the initial velocity of the soccer ball in Q3 if the range of the kick is 20 m.

Horizontally: $s = +20$, $t = 3.0$, $u = \frac{+20}{3.0} \approx +6.67$

Vertically: $a = -9.8$, $v = 0$, $t = 1.5$, *use* $v = u + at$ *to find u.* $0 = u - 9.8 \times 1.5$, $u \approx +14.7$

Initial speed $\approx \sqrt{6.67^2 + 14.7^2} \approx 16$ m s^{-1}. *Direction of velocity is* $\theta°$ *with the horizontal where* $\theta° \approx \tan^{-1}\frac{14.7}{6.67} \approx 66°$

Q5 Calculate the speed of the soccer ball in Q3 at $t = 1.5$ s and $t = 2.0$ s. The ball is kicked at $t = 0$.

Horizontally: $u = +6.67$ *(constant). At* $t = 1.5$, *vertically* $v = 0$.: *the velocity vector is horizontal, and the speed is 6.67* m s^{-1}.

At $t = 2.0$, *vertically* $a = -9.8$, $u \approx +14.7$, *use* $v = u + at$ *to find u.* $v \approx +14.7 - 9.8 \times 2.0 = -4.9$

.: *speed* $\approx \sqrt{6.67^2 + (-4.9)^2} \approx 8.3$ m s^{-1}

Q6 Determine the average velocity and average acceleration of the soccer ball in Q3 during its time of flight.

Average velocity $= \frac{displacement}{time} = \frac{+20}{3.0} = +6.67$ m s^{-1}

Average acceleration = constant acceleration = -9.8 m s^{-2}

Q7 A champagne cork popped out from a bottle at 1.2 m above the floor. It lands on the floor at a horizontal distance of 5.0 m from its starting point 2.0 s later. Calculate the initial velocity of the cork.

Horizontally: $s = 5.0$, $t = 2.0$, $u = \frac{s}{t} = \frac{5.0}{2.0} = 2.5$

Vertically: $s = -1.2$, $a = -9.8$, $t = 2.0$, *use* $s = ut + \frac{1}{2}at^2$ *to find u.* $-1.2 = 2.0u + \frac{1}{2}(-9.8)(2.0)^2$, .: $u = +9.2$

Initial speed $\approx \sqrt{2.5^2 + 9.2^2} \approx 9.5$ *m s^{-1}. Direction of velocity is* $\theta°$ *with the horizontal where* $\theta \approx \tan^{-1}\frac{9.2}{2.5} \approx 75°$

Q8 Use the conservation of energy idea to determine the maximum height reached by the cork in Q7.

Take zero potential energy at the starting pt. At the highest pt, the cork has zero vertical velocity. Let h be the vertical distance above the starting pt.

Total energy at the starting pt = total energy at the highest pt

$\frac{1}{2}m(9.5)^2 + 0 = \frac{1}{2}m(2.5)^2 + m(9.8)h$, .: $h \approx 4.3$

Maximum height above the floor $\approx 1.2 + 4.3 = 5.5$ m

Q9 Use the conservation of energy idea to determine the landing speed of the cork in Q7.

Take zero potential energy at the starting pt. At the highest pt, the cork has zero vertical velocity.

Total energy at the starting pt = total energy at the lowest pt

$\frac{1}{2}m(9.5)^2 + 0 = \frac{1}{2}mv^2 - m(9.8)(1.2)$, $v \approx 11$ m s^{-1}

Q10 Comment on the landing speed of the cork in Q7 if the bottle is tilted at a different angle and the initial speed of the cork remains the same.

The same speed is found using the conservation of energy idea. The tilt angle does not affect the landing speed.

Q11 A monkey on a tall tree branch is 8.0 m above and 25 m horizontally from a hunter's bow and arrow. At what angle should the hunter aim the arrow in order to hit the monkey in the shortest time if the shooting speed of the arrow is 30 ms^{-1}?

Horizontally: $u = +30\cos\theta°$, $s = +25$, $t = \frac{s}{u} = \frac{25}{30\cos\theta°}$

Vertically: $u = +30\sin\theta°$, $s = +8.0$, $a = -9.8$, $t = \frac{s}{u} = \frac{25}{30\cos\theta°}$.

Use $s = ut + \frac{1}{2}at^2$ *to find* θ.

$+8.0 = (+30\sin\theta°)\left(\frac{25}{30\cos\theta°}\right) - 4.9\left(\frac{25}{30\cos\theta°}\right)^2$

$8.0 = 25\tan\theta° - 3.403\sec^2\theta°$

$8.0 = 25\tan\theta° - 3.403(1 + \tan^2\theta°)$, $\tan\theta° \approx 0.4885$, $\theta° \approx 26°$

Q12 A smart hunter aims his arrow directly at a 'stupid' monkey on a tall tree branch some distance away. Hoping to avoid being hit, the monkey drops from the tree branch at the moment the arrow is shot. Describe and explain the fate of the monkey.

The monkey will be hit by the arrow.

The monkey and the arrow have the same downward acceleration of 9.8 m s^{-2}. Both will fall the same distance by the time the arrow reaches the monkey.