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Physics worksheet solutions – Universal gravitation (**$g = 9.8$ N kg^{-1} at the surface of the earth, G** $\approx 6.67\times10^{-11}$ **N.m^{2}kg^{-2}**)

Q1 A 1-kg dumb-bell is at rest on the ground. Calculate the force of gravity exerted by the dumb-bell on the earth.

Force of gravity exerted by the dumb-bell on the earth = force of gravity exerted by the earth on the dumb-bell

$= mg = 1\times9.8 = 9.8$ N

Q2 Calculate the net force exerted by the dumb-bell on the earth in Q1.

The dumb-bell exerts two forces on the earth, force of gravity and reaction force. They have the same magnitude but opposite in direction.

$F_{net} =$ *force of gravity exerted by the dumb-bell on the earth + reaction force of the dumb-bell on the earth* $= 0$

Q3 The centre of the moon is about 385000 km from the centre of the earth. The mass of the moon $= 7.36\times10^{22}$ kg, and the mass of the earth $= 5.98\times10^{24}$ kg. Determine the gravitational field strength of the moon g_{moon} experienced by the earth.

$$g_{moon} = \frac{GM_{moon}}{r^2} = \frac{(6.67\times10^{-11})(7.36\times10^{22})}{(3.85\times10^8)^2} = 3.31\times10^{-5}\text{ N kg}^{-1}$$

Q4 Refer to the information in Q3. Determine the value of each ratio. (i) $\frac{g_{earth}}{g_{moon}}$ and (ii) $\frac{F_{earth.on.moon}}{F_{moon.on.earth}}$, where g stands for gravitational field strength, and F the magnitude of gravitational force.

(i) $\frac{g_{earth}}{g_{moon}} = \frac{M_{earth}}{M_{moon}} = \frac{5.98\times10^{24}}{7.36\times10^{22}} \approx 81$ (ii) $\frac{F_{earth.on.moon}}{F_{moon.on.earth}} = 1$

Q5 There is a point between the earth and the moon where gravitational field due to both bodies is zero. Determine the distance of that point from the centre of the moon.

Let x m *be that distance.*

$$g_{moon} = g_{earth},\ \frac{GM_{moon}}{x^2} = \frac{GM_{earth}}{(3.85\times10^8 - x)^2}$$

$$\frac{(3.85\times10^8 - x)^2}{x^2} = \frac{M_{earth}}{M_{moon}} \approx 81,\ \frac{3.85\times10^8 - x}{x} \approx 9,$$

$10x \approx 3.85\times10^8$, $x \approx 3.85\times10^7$

Q6 A satellite is in a stable circular orbit 250 km above the surface of the earth ($r_{earth} = 6380$ km). Calculate the orbital speed of the satellite.

$$a = g,\ \frac{v^2}{r} = \frac{GM}{r^2},\ v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6380+250)\times10^3}}$$

$v \approx 7.76\times10^3$ m s^{-1}

Note: A satellite at 250 km above the surface of the earth is not high enough to avoid the atmosphere of the earth. The orbit is unstable. The satellite will lose energy and spiral towards the earth. The minimum altitude required is about 320 km.

Q7 Refer to Q6. Describe the effects on the motion of the satellite if the orbital speed is (i) reduced 'slightly', (ii) increased 'slightly', and (iii) increased 'greatly'.

Assume that the satellite is outside the atmosphere of the earth.

(i) *The satellite will follow an elliptical orbit.* (ii) *The satellite will follow an elliptical orbit.* (iii) *It may escape the earth's gravity and keep on moving away from the earth.*

Q8 Determine the orbital radius of a geostationary satellite.

Period $T = 24\times60\times60 = 86400$ s

$$a = g,\ \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2},$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(86400^2)}{4\pi^2}}$$

$\approx 4.23\times10^7$ m

Q9 The circular orbital speeds of Satellite A and Satellite B are v_A and v_B respectively, where $v_B = 2v_A$. Determine the value of each ratio. (i) $\frac{r_B}{r_A}$ and (ii) $\frac{T_B}{T_A}$, where r stands for orbital radius, and T orbital period.

(i) $a = g,\ \frac{v^2}{r} = \frac{GM}{r^2},\ v^2 r = GM$, .: $v_B{}^2 r_B = v_A{}^2 r_A$

$$.:\ \frac{r_B}{r_A} = \frac{v_A{}^2}{v_B{}^2} = \frac{v_A{}^2}{(2v_A)^2} = \frac{1}{4}$$

(ii) $a = g,\ \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2},\ \frac{r^3}{T^2} = \frac{GM}{4\pi^2}$, .: $\frac{r_B{}^3}{T_B{}^2} = \frac{r_A{}^3}{T_A{}^2}$

$$.:\ \frac{T_B{}^2}{T_A{}^2} = \frac{r_B{}^3}{r_A{}^3},\ \frac{T_B}{T_A} = \left(\sqrt{\frac{r_B}{r_A}}\right)^3 = \left(\sqrt{\frac{1}{4}}\right)^3 = \frac{1}{8}$$

Q10 A satellite is 36000 km above the surface of the earth. Estimate the increase in gravitational potential energy per kg of the satellite if its altitude is increased by 1 km.

Distance from the centre of the earth ≈ 42000000 m

$$g = \frac{GM}{r^2} \approx \frac{(6.67\times10^{-11})(5.98\times10^{24})}{(4.2\times10^7)^2} \approx 0.23\text{ N kg}^{-1}$$

There is negligible change in g another 1000 m further.

.: estimated increase in gravitational potential energy per kg $= mg\Delta h = 1\times0.23\times1000 = 230$ J