2003 VCAA Specialist Math Exam 1 Solutions

Part I

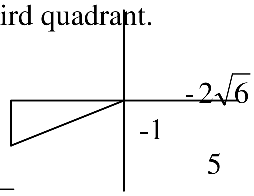
1	2	3	4	5	6	7	8	9	10
D	B	E	A	E	A	D	A	A	C

11	12	13	14	15	16	17	18	19	20
D	E	B	B	B	C	C	E	A	D

21	22	23	24	25	26	27	28	29	30
D	E	C	C	B	C	D	B	D	E

Q1 $\frac{b}{a}=\frac{6}{4}=\frac{3}{2}$ $\therefore$ either C or D. The x-intercepts at $x=2$ and $x=6$. $\therefore$ D.

Q2 x is in the third quadrant.



$\cot x=\frac{A}{O}=\frac{-2\sqrt{6}}{-1}=2\sqrt{6}$. B.

Q3 Sketch $y=\sin^2(2x)$ and $y=\frac{3}{4}$. There are 8 intersections in $0\le x\le 2\pi$. E.

Q4 $\frac{dy}{dx}=\frac{1}{\sqrt{1-\left(\frac{4}{x}\right)^2}}\times\left(\frac{-4}{x^2}\right)=-\frac{4}{x^2\sqrt{1-\left(\frac{4}{x}\right)^2}}$

$=-\frac{4}{x\sqrt{x^2}\sqrt{1-\left(\frac{4}{x}\right)^2}}=-\frac{4}{x\sqrt{x^2-16}}$. A.

Q5 The complex number is in the third quadrant.

$r=\sqrt{3+1}=2$, $\tan\theta=\frac{-1}{-\sqrt{3}}$,

$\theta=\frac{7\pi}{6}$. E.

Q6 $z=\pm\left[4cis\left(\frac{4\pi}{3}\right)\right]^{\frac{1}{2}}=\pm 2cis\left(\frac{2\pi}{3}\right)$,

$z=2\cos\frac{2\pi}{3}+i2\sin\frac{2\pi}{3}=-1+i\sqrt{3}$ or

$z=-2\cos\frac{2\pi}{3}-i2\sin\frac{2\pi}{3}=1-i\sqrt{3}$. A.

Q7 $P(z)=\left(z^3-2z^2\right)+(4z-8)$
$=z^2(z-2)+4(z-2)$
$=(z-2)\left(z^2+4\right)$
$=(z-2)(z-2i)(z+2i)$ D.

Q8 $\left|z-{}^-3\right|=3$, $|z+3|^2=9$,
$(z+3)\left(\overline{z+3}\right)=9$
$(z+3)\left(\bar{z}+3\right)=9$ A.

Note: $\overline{z+3}=\overline{x+yi+3}=\overline{(x+3)+yi}$
$=(x+3)-yi=(x-yi)+3$
$=\bar{z}+3$

Q9 $|z-1|=|z+i|$, $|z-1|=\left|z-{}^-1i\right|$, all the complex numbers that are equidistant from $(1,0)$ and $(0,-1)$.
A.

Q10 $\int_0^{\frac{\pi}{6}}\cos^3(2x)dx=\int_0^{\frac{\pi}{6}}\cos^2(2x)\cos(2x)dx$

$=\int_0^{\frac{\pi}{6}}\left[1-\sin^2(2x)\right]\cos(2x)dx$

$=\int_0^{\frac{\sqrt{3}}{2}}\left(1-u^2\right)\frac{1}{2}du$

$=\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}}\left(1-u^2\right)du$. C.

Q11 When $x=1, y=0$

$x=2, y=\log_e 2$

$x=3, y=\log_e 3$

$x=4, y=\log_e 4$.

Estimated area

$$=\frac{1}{2}(0+\log_e 2)\times 1+\frac{1}{2}(\log_e 2+\log_e 3)\times 1$$

$$+\frac{1}{2}(\log_e 3+\log_e 4)\times 1$$

$$=\log_e 2+\log_e 3+\frac{1}{2}\log_e 4$$

$$=\log_e 12$$ D.

Q12 $\int_0^1\left[2\cos\left(\frac{\pi x}{2}\right)-\left(x^2-1\right)\right]dx$

$$=\left[\frac{4}{\pi}\sin\left(\frac{\pi x}{2}\right)-\frac{x^3}{3}+x\right]_0^1$$

$$=\frac{4}{\pi}+\frac{2}{3}$$ E.

Q13 $\int\frac{3}{x(3-x)}dx=\int\left(\frac{1}{x}+\frac{1}{3-x}\right)dx$

$$=\log_e x-\log_e(3-x)+c$$ B.

Q14 From the graph of the derivative, function *f* has zero gradient at *x* = -2 and 2. At *x* = 2, the point of *f* is an inflection point. At $x<-2$, gradient of *f* is negative. B.

Q15 $f(x)=\int\left(2\sin^2\left(\frac{x}{2}\right)-1\right)dx$

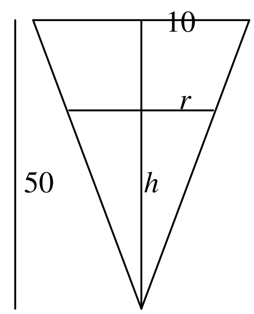
$$=\int(-\cos x)dx$$

$$=-\sin x+c.$$

$$f\left(\frac{\pi}{2}\right)=-\sin\frac{\pi}{2}+c=0,\ \therefore c=1.$$

$$\therefore f(x)=1-\sin x.$$ B.

Q16 $\frac{10}{50}=\frac{r}{h}\quad \therefore r=\frac{h}{5}$



$$V=\frac{1}{3}\pi r^2 h$$

$$\therefore V=\frac{\pi h^3}{75},\ \frac{dV}{dh}=\frac{\pi h^2}{25}.$$

$$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$$

$$-600=\frac{\pi h^2}{25}\times\frac{dh}{dt},\ \therefore \frac{dh}{dt}=-\frac{15000}{\pi h^2}$$ C.

Q17 $x=a$, $y=f(a)$

$x=a+h$, $y\approx f(a)+hf'(a)$.

$x=0$, $y=2$

$x=0.2$, $y\approx 2+0.2\cos 0=2.2$

$x=0.4$, $y\approx 2.2+0.2\cos 0.1$ C.

Q18 Since the four vectors form a quadrilateral, the resultant is zero, i.e.

$$\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}+\boldsymbol{s}=\boldsymbol{0},$$

$$\therefore\ \boldsymbol{p}+\boldsymbol{q}=-\boldsymbol{r}-\boldsymbol{s}$$ E.

Q19 Magnitude of vector $=\sqrt{1^2+2^2+3^2}=\sqrt{14}$.

Unit vector opposite in direction to the given vector is $-\frac{1}{\sqrt{14}}(\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k})=\frac{1}{\sqrt{14}}(-\boldsymbol{i}+2\boldsymbol{j}-3\boldsymbol{k})$.

A.

Q20 $\overrightarrow{OS}=-c\boldsymbol{i}+2c\boldsymbol{j}$, $\overrightarrow{OR}=-2\boldsymbol{i}+\boldsymbol{j}$,

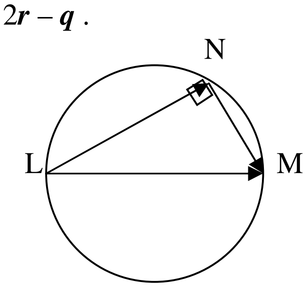
$\overrightarrow{RS}=\overrightarrow{OS}-\overrightarrow{OR}=(-c+2)\boldsymbol{i}+(2c-1)\boldsymbol{j}$,

$\overrightarrow{OS}\bullet\overrightarrow{RS}=-c(-c+2)+2c(2c-1)$

$=5c^2-4c$ D.

Q21 $\angle LNM$ is a right angle. $\overrightarrow{LM}=2\boldsymbol{r}$.

$\overrightarrow{NM}=\overrightarrow{LM}-\overrightarrow{LN}=2\boldsymbol{r}-\boldsymbol{q}$.



$\overrightarrow{NM}\bullet\overrightarrow{LN}=0$

$(2\boldsymbol{r}-\boldsymbol{q})\bullet\boldsymbol{q}=0$

$2\boldsymbol{r}\bullet\boldsymbol{q}-\boldsymbol{q}\bullet\boldsymbol{q}=0$

$2\boldsymbol{r}\bullet\boldsymbol{q}=\boldsymbol{q}\bullet\boldsymbol{q}$ D.

Q22 L is directly north of M when $5t-8=t^2-t$ and $t^2-5t+6>3-t$.

$\therefore t^2-6t+8=0$ and $t^2-4t+3>0$.
$\therefore t=2$ or 4 and $t<1$ or $t>3$.
$\therefore t=4$. E.

Q23 $\boldsymbol{v}=\frac{d}{dt}\boldsymbol{r}=4\boldsymbol{i}-2e^{2t}\boldsymbol{j}$.

When $t=0$, $\boldsymbol{v}=4\boldsymbol{i}-2\boldsymbol{j}$, $\therefore v=\sqrt{4^2+2^2}$
$=\sqrt{20}$. C.

Q24 $\boldsymbol{v}=\int(\cos t\,\boldsymbol{i}-\sin t\,\boldsymbol{j})dt=\sin t\,\boldsymbol{i}+\cos t\,\boldsymbol{j}+\boldsymbol{c}$

When $t=0$, $\boldsymbol{v}=\boldsymbol{i}+\boldsymbol{j}=\sin 0\,\boldsymbol{i}+\cos 0\,\boldsymbol{j}+\boldsymbol{c}$,
$\therefore \boldsymbol{c}=\boldsymbol{i}$ and $\boldsymbol{v}=(\sin t+1)\boldsymbol{i}+\cos t\,\boldsymbol{j}$. C.

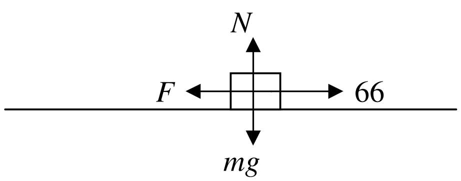
Q25 Let F newtons be the sliding friction.

$\boldsymbol{R}=m\boldsymbol{a}$
$^{+}66+^{-}F=12\times^{+}0.5$
$F=60$

Let $N=mg$ be the normal reaction.

Coefficient of sliding friction $=\frac{F}{N}=\frac{F}{mg}$

$=\frac{60}{12\times 9.8}=0.51$ B.



Q26 C is not correct because both P and Q contribute to balance R.

Q27 Three forces act on the larger mass. They are:

Downward force of gravity on it = Mg
Upward normal reaction force of the ground = R_2
Downward force exerted by the smaller mass = R_1 according to Newton's third law.
D.

Q28 B could not be true because $x^2=t+1$,

$x=\pm\sqrt{t+1}$,

$v=\frac{dx}{dt}=\pm\frac{1}{2\sqrt{t+1}}$,

$a=\frac{dv}{dt}=\pm\frac{1}{4(t+1)^{\frac{3}{2}}}$ which is not constant.

A: $v=t+1$, $a=1$ constant.
C: $x=t^2-1$, $v=2t$, $a=2$ constant.
D: $v^2=x+1$, $a=\frac{d}{dt}\left(\frac{1}{2}v^2\right)=\frac{1}{2}$ constant.
E: $v=t-1$, $a=1$ constant.

Q29 D. Sketch the graph of the function.

Q30 E. Gradient function of $v(t)$.

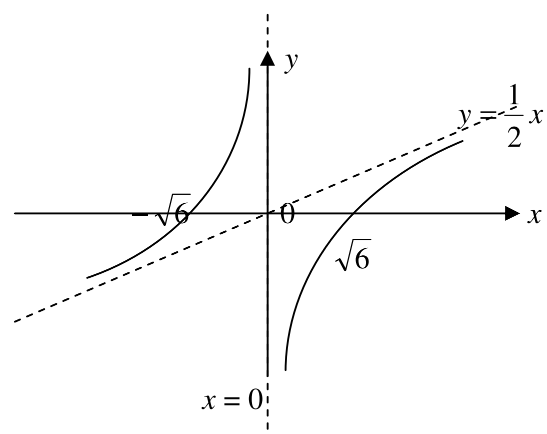
Part II

Q1a $a=\frac{dv}{dt}=4.5-2\sin 2t$

Q1b Resultant force is minimum when acceleration is minimum. This occurs when $\sin 2t=1$ and $a=4.5-2=2.5$.

$R=ma=5\times 2.5=12.5$ newtons.

Q2 $f(x)=\frac{1}{2}x-\frac{3}{x}$



Q3 $y = xe^{3x}$, $\frac{dy}{dx} = e^{3x} + 3xe^{3x}$,

$\frac{d^2y}{dx^2} = 3e^{3x} + 3\left(e^{3x} + 3xe^{3x}\right) = 6e^{3x} + 9xe^{3x}$.

Substitute $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ in the d.e.,

$6e^{3x} + 9xe^{3x} + m\left(e^{3x} + 3xe^{3x}\right) + nxe^{3x} = 0$,

$e^{3x}\left[(6+m) + (9+3m+n)x\right] = 0$.

Since $e^{3x} \neq 0$, $\therefore (6+m) + (9+3m+n)x = 0$,

$\therefore 6 + m = 0$ and $9 + 3m + n = 0$,

$\therefore m = -6$ and $n = 9$.

Q4a Area $= -2 \times \int_0^2 \left(1 - \frac{8}{x^2+4}\right) dx$

$= -2 \times \left[x - 4Tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= -2 \times \left(2 - 4 \times \frac{\pi}{4}\right)$

$= 2(\pi - 2)$.

Q4b $y = 1 - \frac{8}{x^2+4}$, $\frac{8}{x^2+4} = 1 - y$,

$x^2 = \frac{8}{1-y} - 4$.

Volume $= \int_{-1}^{0} \pi x^2 dy = \pi \int_{-1}^{0} \left(\frac{8}{1-y} - 4\right) dy$

$= \pi\left[-8\log_e(1-y) - 4y\right]_{-1}^{0}$

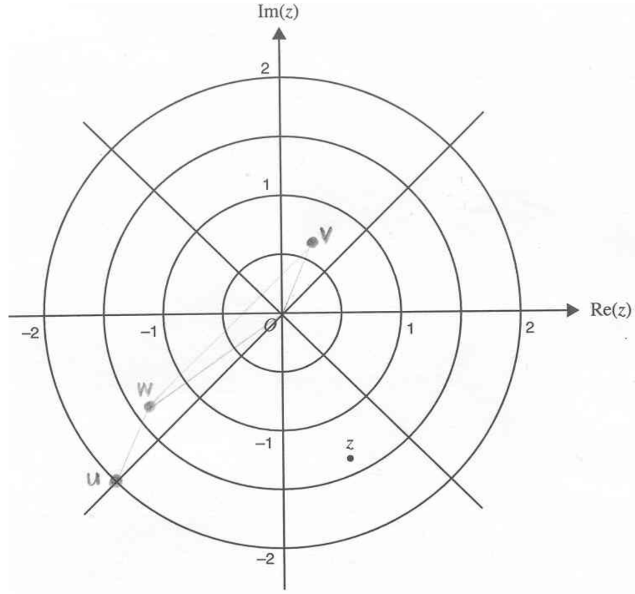
$= \pi(8\log_e 2 - 4)$

$= 4.85$

Q5 (a) $u = z^2 = 2cis(2\theta)$

(b) $v = \frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\sqrt{2}cis(-\theta)}{2}$

(c) $w = u + v$



u and v are added like vectors to give w.

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