2004 VCAA Specialist Math Exam 1 Solutions

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PART I

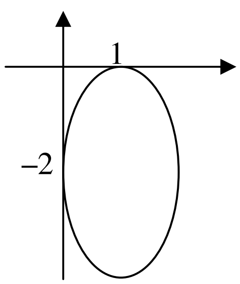
1	2	3	4	5	6	7	8	9	10
E	C	E	D	E	C	B	A	C	D

11	12	13	14	15	16	17	18	19	20
B	A	D	D	C	D	A	D	C	E

21	22	23	24	25	26	27	28	29	30
B	B	D	A	B	A	C	C	E	C

Q1 $y=\frac{-x^2}{2x}+\frac{1}{2x}=-\frac{1}{2}x+\frac{1}{2x}$. Oblique asymptote $y=-\frac{1}{2}x$ and vertical asymptote $x=0$. $\therefore$ E

Q2



$\frac{(x-1)^2}{1^2}+\frac{(y+2)^2}{2^2}=1$, $(x-1)^2+\frac{(y+2)^2}{4}=1$, $\therefore$ C

Q3 $\tan A=\frac{\sin A}{\cos A}$, $\tan A=\frac{1}{\cot A}$,

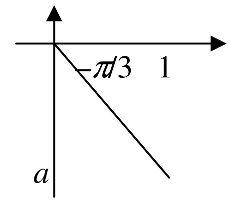
$\cot\left(\frac{\pi}{2}-A\right)=\tan A$, $\tan A=\frac{2\tan\left(\frac{A}{2}\right)}{1-\tan^2\left(\frac{A}{2}\right)}$. $\therefore$ E

Q4 Comparing with the graph of $y=\sec x$, the given graph is the image of $y=\sec x$ under the transformations (in order): reflection in the x-axis; horizontal dilation by a factor of $\frac{1}{2}$, $\therefore a=2$; translation to the right by $\frac{\pi}{4}$, $\therefore$ $b=\frac{\pi}{4}$. $\therefore$ D

Q5 $\frac{1}{z}+\frac{1}{\bar{z}}=\frac{\bar{z}+z}{z\bar{z}}=\frac{2x}{x^2+y^2}$ is a real number. $\therefore$ E

Q6 $\tan\left(-\frac{\pi}{3}\right)=\frac{a}{1}$, $a=-\sqrt{3}$.

$\therefore$ C



Q7 Degree 4, $\therefore$ 4 roots (fundamental theorem of algebra). Real coefficients, according to the conjugate roots theorem, the complex roots come in pairs (conjugate pairs) or there are no complex roots at all. $\therefore$ B

Q8 $w^{-1}=\frac{1}{w}=\frac{\bar{w}}{w\bar{w}}=\frac{\bar{w}}{|w|^2}=\frac{\bar{w}}{2.25}$. Point S represents $\bar{w}$ and point P represents w^{-1}. $\therefore$ A

Q9 The shaded region is a set of complex numbers with $\frac{\pi}{3}<Arg(z)<\frac{\pi}{2}$. $\therefore$ C

Q10 $\int\frac{1}{x^2+16}dx=\frac{1}{4}\int\frac{4}{x^2+4^2}dx=\frac{1}{4}Tan^{-1}\left(\frac{x}{4}\right)+c$

$\therefore$ D

Q11 $\int_0^a\left(\sin^2\left(\frac{3x}{2}\right)-\cos^2\left(\frac{3x}{2}\right)\right)dx$

$=-\int_0^a\left(\cos^2\left(\frac{3x}{2}\right)-\sin^2\left(\frac{3x}{2}\right)\right)dx=-\int_0^a\cos(3x)dx$

$=-\left[\frac{\sin(3x)}{3}\right]_0^a=-\frac{\sin(3a)}{3}$. $\therefore$ B

Q12 $\int_0^{\frac{\pi}{3}}\cos^2(x)\sin^3(x)dx$

$=\int_0^{\frac{\pi}{3}}\cos^2(x)\sin^2(x)\sin(x)dx$

$=\int_0^{\frac{\pi}{3}}\cos^2(x)\left(1-\cos^2(x)\right)\sin(x)dx$ Let $u=\cos(x)$

$-du=\sin(x)dx$

$=\int_1^{\frac{1}{2}}-u^2\left(1-u^2\right)du$

$=\int_{\frac{1}{2}}^1u^2\left(1-u^2\right)du$. $\therefore$ A

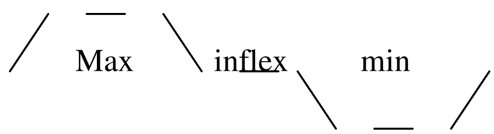
Q13 $\int\frac{2}{(3-x)^2}-\frac{1}{3-x}dx=\frac{2}{3-x}+\log_e(3-x)+c$. $\therefore$ D

Q14 $F(x)=\int f(x)dx\ \therefore \dfrac{dF(x)}{dx}=f(x)$.

The stationary points are at (where $\dfrac{dF(x)}{dx}=f(x)=0$)

$x=\dfrac{\pi}{2},\pi$ and $\dfrac{3\pi}{2}$.

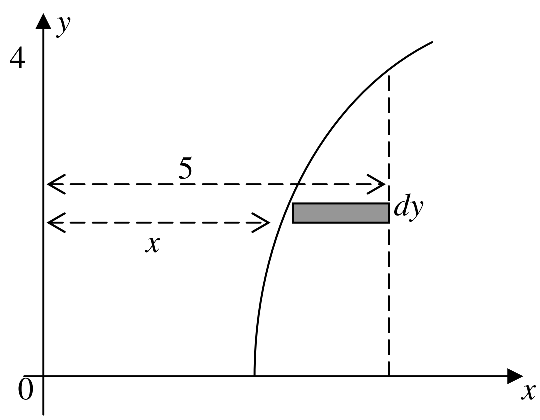
x		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$	
$\frac{dF(x)}{dx}$	+	0	–	0	–	0	+



$\therefore$ D

Q15 At $x=3.5$, $y=1.803$, at $x=4.5$, $y=3.354$.
Estimated area $\approx 1.803\times 1+3.354\times 1=5.16\ \therefore$ C

Q16



$y=\sqrt{x^2-9}$, $\therefore x^2=y^2+9$.

Vol of solid $=\int_0^4 \pi 5^2-\pi x^2 dy$

$=\pi\int_0^4 25-(y^2+9)dy=\pi\int_0^4(16-y^2)dy$. $\therefore$ D

Q17 $2^2+x^2+3^2=4^2$, $\therefore x=\sqrt{3}$. Since $\boldsymbol{p}$ and $\boldsymbol{q}$ are parallel, $\therefore \boldsymbol{q}=m\boldsymbol{p}$.
Hence $-4\boldsymbol{i}+y\boldsymbol{j}-6\boldsymbol{k}=m(2\boldsymbol{i}+x\boldsymbol{j}+3\boldsymbol{k})$.
$\therefore 2m=-4,\ m=-2$,
$y=mx=-2\sqrt{3}$. $\therefore$ A

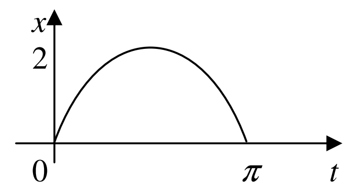
Q18 $\boldsymbol{u}\bullet(\boldsymbol{u}-2\boldsymbol{v})=\boldsymbol{u}\bullet\boldsymbol{u}-2\boldsymbol{u}\bullet\boldsymbol{v}$
$=3^2+(-4)^2+5^2-2(3\times 2-4\times 3-5\times 1)$
$=72$. $\therefore$ D

Q19 $\boldsymbol{b}\bullet(\boldsymbol{a}-\boldsymbol{b})=\boldsymbol{b}\bullet\boldsymbol{c}=0$ because $\boldsymbol{b}\perp\boldsymbol{c}$. $\therefore$ C

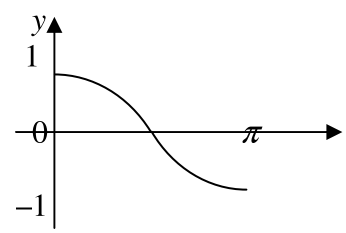
Q20 Parametric equations are $x=2\sin(t)$, $y=\cos(t)$.

Since $\sin^2(t)+\cos^2(t)=1$, $\left(\dfrac{x}{2}\right)^2+y^2=1$,

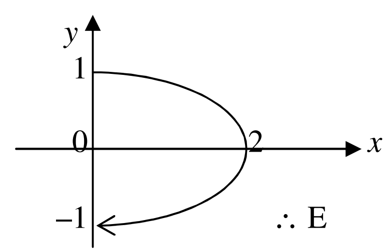
$\therefore \dfrac{x^2}{4}+y^2=1$. Since $0\le t\le\pi$,



$\therefore 0\le x\le 2$,



$\therefore -1\le y\le 1$ and cannot be B.
The path of the particle is shown below.



$\therefore$ E

Q21 $\boldsymbol{r}(t)=\int 3\sin(2t)\boldsymbol{i}+4\boldsymbol{j}\ dt$,

$\boldsymbol{r}(t)=-\dfrac{3}{2}\cos(2t)\boldsymbol{i}+4t\boldsymbol{j}+\boldsymbol{c}$

$\boldsymbol{r}(0)=-\dfrac{3}{2}\cos(0)\boldsymbol{i}+0\boldsymbol{j}+\boldsymbol{c}=\dfrac{3}{2}\boldsymbol{i}$, $\therefore \boldsymbol{c}=3\boldsymbol{i}$.

Hence $\boldsymbol{r}(t)=\left(3-\dfrac{3}{2}\cos(2t)\right)\boldsymbol{i}+4t\boldsymbol{j}$. $\therefore$ B

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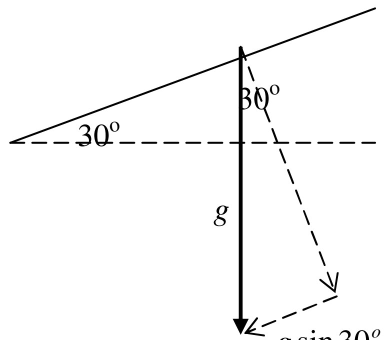
Q22

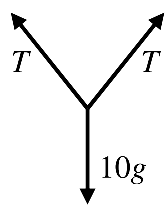


$g\sin 30^o = \frac{g}{2}$ $\therefore$ B

Q23 $^+2T\cos 30^o + ^- 10g = 0$,

$T = \frac{10g}{\sqrt{3}} = \frac{10\sqrt{3}g}{3}$

$\therefore$ D



Q24 Resultant force = $\boldsymbol{R} + \boldsymbol{S} + \boldsymbol{T}$
= $(2\boldsymbol{i} + \boldsymbol{j}) + (\boldsymbol{i} + 10\boldsymbol{j}) + (3\boldsymbol{i} - 3\boldsymbol{j}) = 6\boldsymbol{i} + 8\boldsymbol{j}$

$\therefore m\boldsymbol{a} = 6\boldsymbol{i} + 8\boldsymbol{j}$, $\boldsymbol{a} = \frac{1}{m}(6\boldsymbol{i} + 8\boldsymbol{j}) = \frac{1}{5}(6\boldsymbol{i} + 8\boldsymbol{j})$.

$|\boldsymbol{a}| = \frac{1}{5}\sqrt{6^2 + 8^2} = 2$, $\therefore$ A

Q25 $u = ^+21$, $a = ^-9.8$, $t = 10$, $s = ^-h$

$s = ut + \frac{1}{2}at^2$, $^-h = ^+21\times 10 + \frac{1}{2}(^-9.8)\times 10^2$

$\therefore h = 280$, $\therefore$ B

Q26 Since the body is on the point of sliding down, the friction force points up the plane. $\therefore$ A

Q27 Velocity v is a vector quantity. In one dimension (straight line motion) the +/– signs indicate the direction (e.g. forward/backward) of motion. 'the minimum value of v' makes sense when it refers to the minimum speed of the particle.

$v = \dot{x} = 2.5 - \frac{9}{2}\sin\left(\frac{t}{2}\right)$, $\therefore ^-2 \le v \le ^+7$, but the speed is $|v|$ where $0 \le |v| \le 7$. $\therefore$ C

Note: Some may disagree with the writer's interpretation.

Q28 $y = \sin(2x)$, $\frac{dy}{dx} = 2\cos(2x)$, $\frac{d^2y}{dx^2} = -4\sin(2x)$

$\therefore$ C

Q29 Excess in temperature is $(y - 4)$. At $t = 0$, $y = 20$.
$\therefore$ E

Q30 $a = 16x$, $\frac{d\left(\frac{1}{2}v^2\right)}{dx} = 16x$, $\frac{1}{2}v^2 = \int 16x\,dx$,

$\frac{1}{2}v^2 = 8x^2 + c$. When $x = 0$, $v = -5$, $\therefore$ $c = \frac{25}{2}$.

Hence $v^2 = 16x^2 + 25$ and $v = -\sqrt{16x^2 + 25}$ satisfies the requirement $x = 0$, $v = -5$. $\therefore$ C

PART II

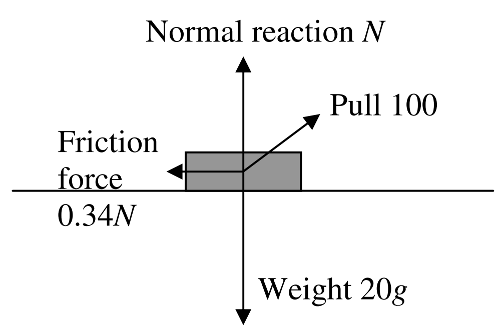
Q1a. Let $u = \sqrt{2x}$, $\frac{d}{dx}\left(Sin^{-1}\left(\sqrt{2x}\right)\right) = \frac{du}{dx}\frac{d}{du}\left(Sin^{-1}(u)\right)$

$= \frac{1}{\sqrt{2x}} \times \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{2x}} \times \frac{1}{\sqrt{1-2x}} = \frac{1}{\sqrt{2x(1-2x)}}$.

Q1b. $\int_{\frac{1}{8}}^{\frac{1}{4}} \frac{1}{\sqrt{2x(1-2x)}}dx = \left[Sin^{-1}\left(\sqrt{2x}\right)\right]_{\frac{1}{8}}^{\frac{1}{4}}$

$= Sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - Sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$.

Q2a.



Q2b. Vertical component:
$^+N + ^+100\sin 40^o + ^- 20g = 0$ (1)
Horizontal component:
$^+100\cos 40^o + ^- 0.34N = 20a$ (2)
From (1), $N = -100\sin 40^o + 20g$, sub. in (2),
$a = 1.59$.

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Q3 $f(x)=\int 15x\sqrt{2-x}dx$, let $u=2-x$, $\therefore x=2-u$ and $dx=-du$.

$f(x)=\int 15(2-u)\sqrt{u}(-du)=\int\left(-30u^{\frac{1}{2}}+15u^{\frac{3}{2}}\right)du$

$=\frac{-30u^{\frac{3}{2}}}{\frac{3}{2}}+\frac{15u^{\frac{5}{2}}}{\frac{5}{2}}+c=-20(2-x)^{\frac{3}{2}}+6(2-x)^{\frac{5}{2}}+c$.

Since $f(2)=0$, $\therefore c=0$.

$f(x)=-20(2-x)^{\frac{3}{2}}+6(2-x)^{\frac{5}{2}}=(2-x)^{\frac{3}{2}}[-20+6(2-x)]$

$=(-6x-8)(2-x)^{\frac{3}{2}}$. $\therefore a=-6$ and $b=-8$.

Q4a. Let (^***b***) be a unit vector in the direction of ***b***.

$(\hat{\boldsymbol{b}})=\frac{1}{b}\boldsymbol{b}=\frac{1}{\sqrt{4^2+(-3)^2}}(4\boldsymbol{i}-3\boldsymbol{j})=\frac{1}{5}(4\boldsymbol{i}-3\boldsymbol{j})$

Scalar resolute of ***a*** in the direction of ***b*** = ***a***•(^***b***)

$=\frac{1}{5}(6\times 4-2\times 3)=\frac{18}{5}$

Q4b. $\boldsymbol{AP}=\boldsymbol{OP}-\boldsymbol{OA}=\frac{18}{5}(\hat{\boldsymbol{b}})-\boldsymbol{a}$

$=\frac{18}{5}\times\frac{1}{5}(4\boldsymbol{i}-3\boldsymbol{j})-(6\boldsymbol{i}+2\boldsymbol{j})$

$=-\frac{78}{25}\boldsymbol{i}-\frac{104}{25}\boldsymbol{j}$

$=\frac{26}{25}(-3\boldsymbol{i}-4\boldsymbol{j})$,

$\therefore |\boldsymbol{AP}|=\frac{26}{25}\times\sqrt{(-3)^2+(-4)^2}=\frac{26}{5}$.

Q5a. w is in the third quadrant of the complex plane,

$Arg\ w=Tan^{-1}\left(\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}\right)=Tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=-\frac{5\pi}{6}$,

$|w|=\sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(-\frac{1}{2}\right)^2}=1$. $\therefore w=cis\left(-\frac{5\pi}{6}\right)$.

Q5b. $w^k=1$, $\left[cis\left(-\frac{5\pi}{6}\right)\right]^k=1$, $cis\left(-\frac{5k\pi}{6}\right)=1$,

i.e. $\cos\left(-\frac{5k\pi}{6}\right)+i\sin\left(-\frac{5k\pi}{6}\right)=1$,

hence $\cos\left(-\frac{5k\pi}{6}\right)=1$ and $\sin\left(-\frac{5k\pi}{6}\right)=0$,

$\therefore$ the smallest positive integer k that satisfies both equations is 12.

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