Solutions to the VCAA 2010 Mathematical Methods (CAS) Examination 1 sample questions

(Note: This is a collection of sample questions and not a sample examination)

Q1 $x^3+(k+1)x^2+kx=0$

$x\left(x^2+(k+1)x+k\right)=0$

$x=0$ or $x=\dfrac{-(k+1)\pm\sqrt{(k+1)^2-4k}}{2}=\dfrac{-(k+1)\pm\sqrt{(k-1)^2}}{2}$

$=-k$ or -1

$\therefore x\in\{-k,-1,0\}$

Q2 Solve $2=na$ and $a+1=3n$ simultaneously to find a.

$n=\dfrac{2}{a}$, $\therefore a+1=\dfrac{6}{a}$

$a^2+a-6=0$

$(a+3)(a-2)=0$

$a=-3$ or 2

Q3 Let (x',y') be the image of the point (x,y) under the transformation defined by $\begin{bmatrix}2 & 0\\ 0 & -3\end{bmatrix}$.

$\begin{bmatrix}x'\\ y'\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & -3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

$\begin{bmatrix}x'\\ y'\end{bmatrix}=\begin{bmatrix}2x\\ -3y\end{bmatrix}$

$\therefore x=\dfrac{x'}{2}$ and $y=-\dfrac{y'}{3}$

Equation of the image of $y=\dfrac{1}{x}$ is $-\dfrac{y'}{3}=\dfrac{2}{x'}$

$y'=-\dfrac{6}{x'}$ or $y=-\dfrac{6}{x}$

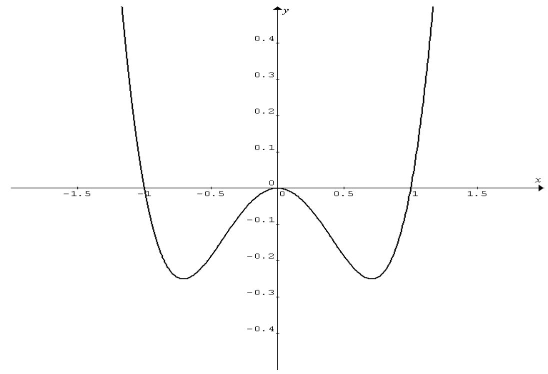
Sequence of transformations:
- reflection in the x-axis
- dilation from the x-axis by a factor of 3
- dilation from the y-axis br a factor of 2

Q4 $f(x)=x^4-x^2=x^2(x+1)(x-1)$

x-intercepts: -1, 0, 1

$f'(x)=4x^3-2x=2x\left(\sqrt{2}x+1\right)\left(\sqrt{2}x-1\right)$

Stationary points: $x=-\dfrac{\sqrt{2}}{2}$, 0, $\dfrac{\sqrt{2}}{2}$



The function is strictly decreasing on the intervals $\left(-\infty,-\dfrac{\sqrt{2}}{2}\right]$ and $\left[0,\dfrac{\sqrt{2}}{2}\right]$.

Note: The answer $\left(-\infty,-\dfrac{\sqrt{2}}{2}\right]\cup\left[0,\dfrac{\sqrt{2}}{2}\right]$ given in the original VCAA document is different from the answer above.

The function is **not** strictly decreasing on the set $\left(-\infty,-\dfrac{\sqrt{2}}{2}\right]\cup\left[0,\dfrac{\sqrt{2}}{2}\right]$ because $a<b$ does not imply $f(a)>f(b)$ for some a and b in the set, e.g. when $a=-0.9$ and $b=0$, $f(a)<f(b)$.

Q5 $\sin(x)+\cos(x)=0$

$\dfrac{1}{\cos(x)}(\sin(x)+\cos(x))=0$, where $\cos(x)\neq 0$

$\dfrac{\sin(x)}{\cos(x)}+1=0$

$\tan(x)=-1$

$\therefore x=\dfrac{3\pi}{4}+n\pi$, where $n\in Z$

Q6

$mx+12y=12$

$3x+my=m$

$\therefore y=-\dfrac{m}{12}x+1$ and $y=-\dfrac{3}{m}x+1$ have

(i) a unique solution when $-\dfrac{m}{12}\neq-\dfrac{3}{m}$

$m^2\neq 36$, $m\neq\pm 6$

(ii) infinitely many solutions when $-\dfrac{m}{12}=-\dfrac{3}{m}$

$m^2=36$, $m=\pm 6$

Q7

The transition matrix is $\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}$.

In the long term the percentage of successful attempts does not depend on the outcome of the last attempt, i.e. the two columns in $\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}^n$ become the same.

$\begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}^n \to \begin{bmatrix} p & p \\ 1-p & 1-p \end{bmatrix}$ as $n \to \infty$

$\therefore \begin{bmatrix} 0.84 & 0.64 \\ 0.16 & 0.36 \end{bmatrix}\begin{bmatrix} p & p \\ 1-p & 1-p \end{bmatrix} = \begin{bmatrix} p & p \\ 1-p & 1-p \end{bmatrix}$

$\therefore 0.84p + 0.64(1-p) = p$

$\therefore p = 0.8$

In the long term, 80% of her attempts are successful.

Q8 $h(x) = \frac{x^n}{e^x}$

$h'(x) = \frac{e^x nx^{n-1} - e^x x^n}{(e^x)^2} = \frac{nx^{n-1} - x^n}{e^x}$

Let $h'(x) = 0$

$\therefore nx^{n-1} - x^n = 0$, $x^{n-1}(n-x) = 0$, $x = 0$ or n

x	$< n$	n	$> n$
$h'(x)$	+	0	–

It is local maximum at $x = n$.

Q9 $g(x) = x^2$

$g(u+v) + g(u-v) = (u+v)^2 + (u-v)^2$

$= 2(u^2 + v^2) = 2(g(u) + g(v))$

Q10 $f(x) = e^x + e^{-x}$, $g(x) = e^x - e^{-x}$

(i) $[f(x)]^2 = (e^x + e^{-x})^2 = e^{2x} + e^{-2x} + 2 = f(2x) + 2$

(ii) $f(x)g(x) = (e^x + e^{-x})(e^x - e^{-x}) = e^{2x} - e^{-2x} = g(2x)$

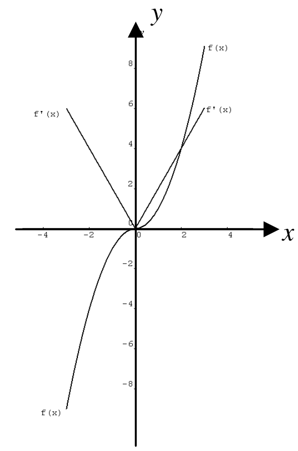
(iii) $[f(x)]^2 - [g(x)]^2 = (f(x) - g(x))(f(x) + g(x))$

$= 2e^{-x} \times 2e^x = 4$

Q11 The average value of $y = e^x$ over $[0,2]$ is

$\frac{\int_0^2 e^x dx}{2-0} = \frac{[e^x]_0^2}{2} = \frac{e^2 - 1}{2}$.

Q12a.



b. $f'(x) = \begin{cases} -2x, x \le 0 \\ 2x, \ x > 0 \end{cases}$ or $f'(x) = 2x\,\mathrm{sgn}(x)$ or $f'(x) = |2x|$

Q13a. $v(0) = 24$

b. $\frac{24}{t+1} < 2$, $\frac{t+1}{24} > \frac{1}{2}$, $\therefore t > 11$ s

c. Distance $= \int_0^{10} \frac{24}{t+1} dt = [24\log_e(t+1)]_0^{10} = 24\log_e 11$ metres

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors

