VCAA Further Mathematics Exam 2 Solutions 2005
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Core

Q1a the average value

Q1bi $r^2 = 0.9058$, $\therefore r = -\sqrt{0.9058} = -0.952$.

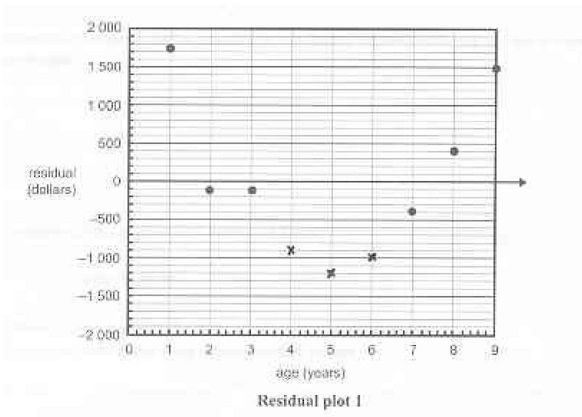
Q1bii 90.58%

Q1c From graph: vertical axis intercept at 17500,

$\text{gradient} = \frac{14000-17500}{3-0} = -1200$.

$\therefore value = [17500] + [^-1200] \times age$.

Q1d



Q1e Yes, it has a definite curved pattern.

Q1f $\log(9) = 0.95$

Q1g Negative, linear and strong.

Q1h $value = 18300 - 10800 \times \log(3) = 13147 \approx 13100$

Q1i The residual plot has a random pattern.

Q1j $\log(value)$ or $\sqrt{value}$ compresses the vertical scale.

Module 1: Number patterns and applications

Q1a There is an increase of 0.3 everyday. 4.2, 4.5, 4.8, 5.1, 5.4
Distance on Day 5 = 5.4 km

Q1b $t_n = a + (n-1)d > 7$; $4.2 + (n-1) \times 0.3 > 7$

$(n-1)0.3 > 2.8$; $n - 1 > 9.33$; $\therefore n > 10.33$, $\therefore$ Day 11.

Q1c $S_2 = 4.2 + 4.5 = 8.7$;

$S_{12} = \frac{n}{2}(2a + (n-1)d) = \frac{12}{2}(2 \times 4.2 + 11 \times 0.3) = 70.2$

Required distance $= 70.2 - 8.7 = 61.5$ km.

Q1d $t_n = 4.2 + (n-1) \times 0.3 = 0.3n + b$, $\therefore 0.3n + 3.9 = 0.3n + b$, $\therefore b = 3.9$

Q2a No common difference because $3.5 - 3 \neq 4.05 - 3.5$, $\therefore$ not arithmetic. No common ratio because $\frac{3.5}{3} \neq \frac{4.05}{3.5}$, $\therefore$ not geometric.

Q2b $c = w_{n+1} - 1.1w_n = w_2 - 1.1w_1 = 3.5 - 1.1 \times 3 = 0.2$

Q2c $w_4 = 1.1w_3 + 0.2 = 1.1 \times 4.05 + 0.2 = 4.655$,

$w_5 = 1.1w_4 + 0.2 = 1.1 \times 4.655 + 0.2 = 5.3205 \approx 5.32$ km.

Q3a 5% greater than means 105% times of.
Distance travelled in Day 2 $= 105\% \, of \, 10 = 1.05 \times 10 = 10.5$ km.

Q3b $a = 10$, $r = 1.05$, $n = 14$.

$S_n = \frac{a(r^n - 1)}{r - 1}$, $S_{14} = \frac{10(1.05^{14} - 1)}{1.05 - 1} = 196$ km.

Q3ci Run : walk : total = 3 : 2 : 5
Walk : total = w : 10 = 2 : 5. $\therefore w = 4$ km.

Q3cii Run : walk = 25% : 75% = 25 : 75 = 1 : 3

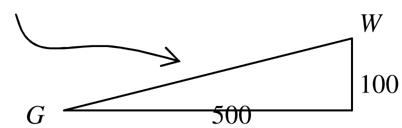
Q3di $t_n = 0.90t_{n-1} + 1.2$, where $t_1 = 5$ and $n = 2,3,4,......$

Q3dii Suppose $t_n > 12$, then $t_n = 0.90t_{n-1} + 1.2 > 12$, $\therefore t_{n-1} > 12$, i.e. the one before is also greater than 12. The same argument leads to all previous days are greater than 12 and this contradicts the fact that on the first day she travels 5 km.

Module 2: Geometry and trigonometry

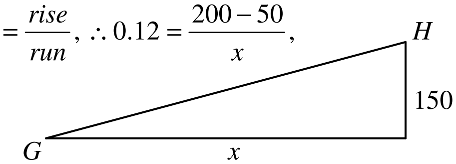
Q1a $\text{Difference} = 200 - 150 = 50$ m

Q1b $\text{Shortest length} = \sqrt{500^2 + (150-50)^2} = 510$ m



Q1c $Average\ slope = \frac{rise}{run}$, $\therefore 0.12 = \frac{200-50}{x}$,

$x = \frac{150}{0.12} = 1250$ m.



Q1d 2 km = 200000 cm. x cm : 200000 cm = 1 : 40000,

$\frac{x}{200000} = \frac{1}{40000}$, $\therefore x = 5$.

Q2ai $2.1\cos 40^o = 1.61 \approx 1.6$ km

Q2aii $1.7\cos 75^o = 0.44$ km

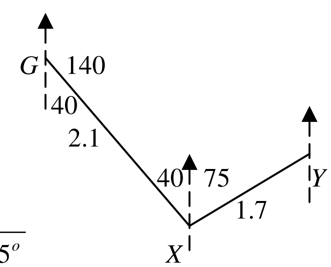
$1.61 - 0.44 = 1.17 \approx 1.2$ km

Q2b $\angle GXY = 40 + 75 = 115^o$.

Q2c The cosine rule: GY

$= \sqrt{2.1^2 + 1.7^2 - 2(2.1)(1.7)\cos 115^o}$

$= 3.212 \approx 3.2$ km



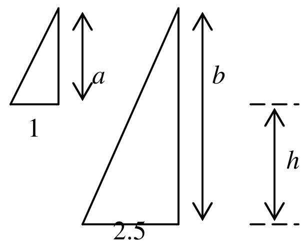
Q2d The sine rule: $\frac{\sin \angle XGY}{1.7} = \frac{\sin 115^o}{3.212}$,

$\angle XGY = 28.66^o$, bearing of Y from $G = 140 - 29 = 111^o$.

Q3a $\frac{1}{a} = \tan 25^o$, $a = 2.1445$

$\frac{2.5}{b} = \tan 25^o$, $b = 5.3613$

$\therefore h = b - a = 3.2$ m



Q3b Length ratio is 2 : 5

$\therefore$ area ratio is $2^2 : 5^2 = 4 : 25$

Let d be the length of the hut.

$\therefore storage : V = 4d : 25d$

$\therefore storage = \frac{4d}{25d} V = \frac{4}{25} V$.

The fraction is $\frac{4}{25}$.

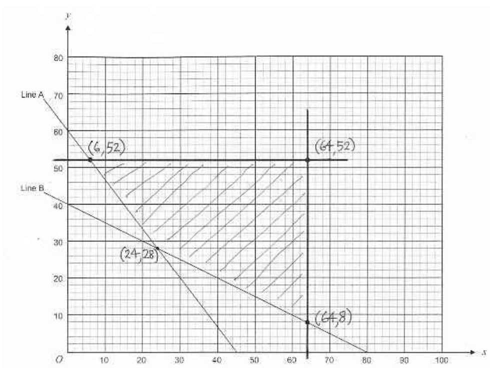
Module 3: Graphs and relations

Q1a Find the x and y-intercepts: $x = 45,\ y = 60$, $\therefore$ line A

Q1b Read from graph 1, $(24,28)$.

Q1c $x \le 64,\ y \le 52$.

Q1d



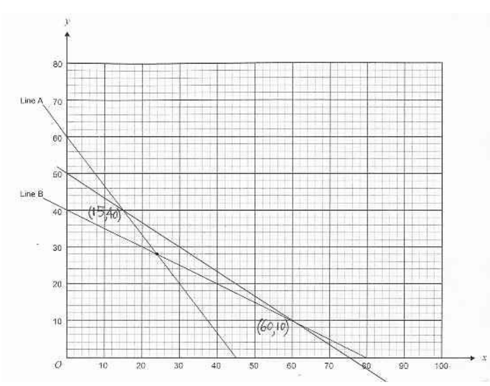
Q1e $C = 100x + 70y$

Q1f Point $(6,52)$ gives the lowest total cost

$= 100(6) + 70(52) = 4240$ dollars

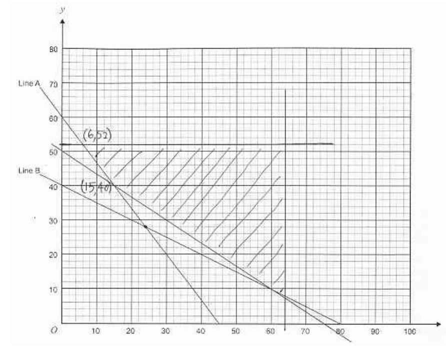
Q1g $2x + 3y \ge 150$.

Q1h



Q1i $2x + 3y = 2(25) + 3(32) = 146$ $\therefore$ less than the required number of 150 or more.

Q1ji



$C = 120x + 90y$, it has the same gradient as line A.

When it moves away from the origin towards the feasible region, it first comes in contact with the feasible region along line A between point $(6,52)$ and point $(15,40)$. Any point along line A between those two points gives the minimum total cost.

Minimum cost $= 120(6) + 90(52) = 5400$ dollars.

Q1jii The number of hours per week that each stand can operate to minimise total cost must satisfy the equation $120x + 90y = 5400$, i.e. $4x + 3y = 180$, where $6 \le x \le 15$.

Module 4: Business-related mathematics

Q1a Interest $= 4560 - 4000 = \$560$

Q1b $P = 4000 - 500 = 3500$, $r = \frac{560 \times 100}{2 \times 3500} = 8$, $\therefore$ 8% pa.

Q1c Effective rate $\approx \frac{2n}{n+1} \times$ flat rate

$= \frac{2(24)}{24+1} \times 8 = 15.4$. $\therefore$ 15.4%

Q1d For flat interest rate, interest is still charged on the amount that is repaid. $\therefore$ effectively the interest rate on the amount owing is higher than the flat rate.

Q2a Depreciation $= 4000 - 1000 = \$3000$

Flat rate of depreciation $= \frac{3000 \times 100}{4000 \times 5} = 15$, $\therefore$ 15% pa

Q2b $V = P\left(1 - \frac{r}{100}\right)^n$, $1000 = 4000\left(1 - \frac{r}{100}\right)^5$,

$r = 24.2$, $\therefore$ 24.2% pa

Q3ai $P = 10000,\ R = 1 + \frac{4.8}{12 \times 100} = 1.004,\ n = 5 \times 12 = 60$.

Q3aii $A = PR^n = 10000 \times 1.004^{60} = \$12706.41$

Q3b $A = PR^n + \frac{Q(R^n - 1)}{R - 1} = 4000 \times 1.004^{60} + \frac{100(1.004^{60} - 1)}{1.004 - 1}$

$= \$11848.58$

Q3ci First two years:

$A = PR^n + \frac{Q(R^n - 1)}{R - 1} = 4000 \times 1.004^{24} + \frac{100(1.004^{24} - 1)}{1.004 - 1} = 6915.90$

Next three years: $A = PR^n + \frac{Q(R^n - 1)}{R - 1}$,

$13000 = 6915.90 \times 1.004^{36} + \frac{Q(1.004^{36} - 1)}{1.004 - 1}$, $Q = \$129.80$ is the new amount.

Q3cii Total interest earned

$= 13000 - (4000 + 100 \times 24 + 129.80 \times 36) = \$1927.20$

Module 5: Networks and decision mathematics

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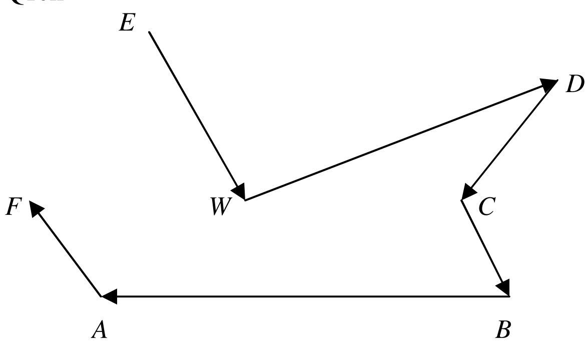
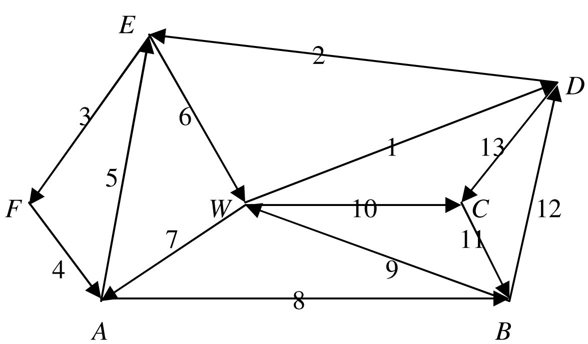
Q1a Number of edges connected to W is 5, $\therefore$ degree 5.
Q1b The route taken must be $B \to C \to D \to C \to W$.
Minimum distance $= 6+6+6+12 = 30$ km.
Q1ci It is a Hamilton path.
Q1cii



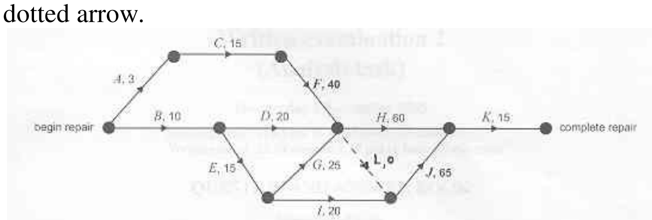
Q1di An Euler circuit exists if every vertex has even degree. The given network has two odd-degree vertices, $\therefore$ the planned journey is impossible.

Q1dii An Euler path is one that uses each edge exactly once. The following diagram shows an Euler path for the network. The edges are numbered to show the order. The Euler path finishes at vertex C.



Q2a F, D, G   50
$10+15+25=50$

Q2b Minimum time $= 50+60+15 = 125$ minutes.
Q2ci Because it is not part of the critical path.
Q2cii EST for F is 3 +15 =18. EST for H is 50. $\therefore$ maximum duration for F is 50 − 18 = 32 minutes.
Q2di Introduce a dummy activity L of 0 min duration, shown as a dotted arrow.



Q2dii LST (H) = EST (K) − duration of H
$= (3+15+40+60) - 60 = 58$ minutes.
Q2diii Minimum time needed $= 58+60+15 = 133$ minutes.
Q2div Activity D. It can be delayed by 28 minutes.