Review of basic electricity

Electric current *I* *is defined as the amount of charge Q passing through in a unit of time t.*

$$I = \frac{Q}{t} \text{ or } Q = It$$

An ampere (A) is a unit of current for the passage of a coulomb (C) of charge in a second (s), 1 A ≡ 1 Cs^{-1}.

*In an electric circuit the current through a component is measured with an **ammeter** connected in series with it.*

Potential *V* *at a point is the amount of electric potential energy E possessed by each unit of charge at that point.*

$$V = \frac{E}{Q} \text{ or } E = VQ$$

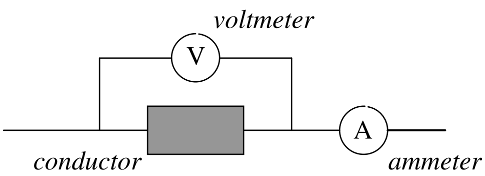
The unit to measure potential is volt (V), it is defined as joule (J) per coulomb (C), 1 V ≡ 1 JC^{-1}

Potential difference, *also denoted as V and measured in V is the difference in potential between two points. When current flows from high to low potential, electric potential energy of the charges changes to other forms of energy. The amount of energy change is also given by $E = VQ$, where V is the potential difference measured with a **voltmeter** connected to the two points.*

$\therefore$ $E = VIt$, and power dissipated $P = \frac{E}{t} = VI$.

Resistance *R* (Ω) *of a conductor is a measure of the ability of the conductor in restricting the flow of electric current, and it is defined as the ratio of potential difference V (V) to current I (A).*

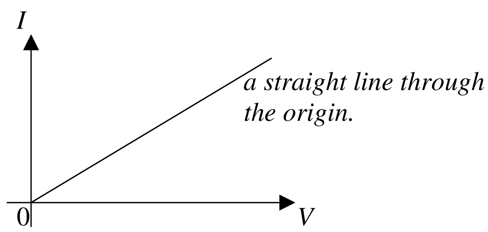
$$R = \frac{V}{I}$$



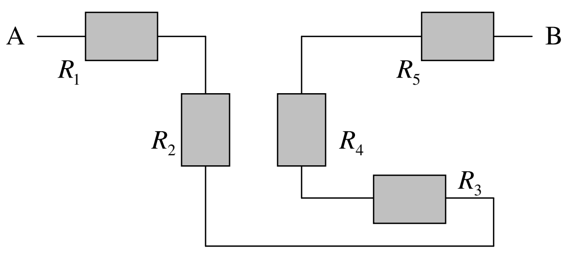
Ohm's law

Ohm's law states that for some conductors the resistance stays constant when potential difference and current vary.

*Conductors that obey Ohm's law are called **ohmic conductors** (**resistors**) and have the following I-V characteristics.*

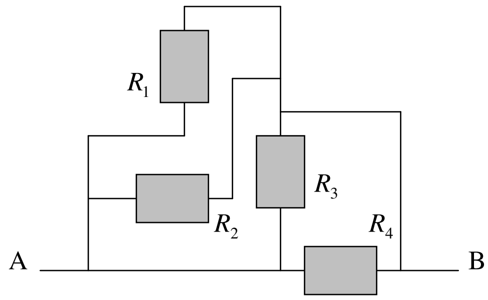


Components connected in series



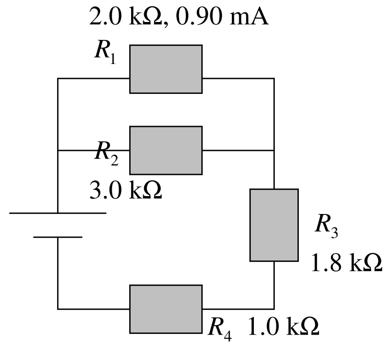
1) $I = I_1 = I_2 = I_3 = ...$
2) $V_{AB} = V_1 + V_2 + V_3 + ...$
3) $R_T = R_1 + R_2 + R_3 + ...$ *remains constant if the components are ohmic resistors. Also* $R_T = \frac{V_{AB}}{I}$.
4) $\frac{V_1}{V_2} = \frac{R_1}{R_2}$, $\frac{V_2}{V_3} = \frac{R_2}{R_3}$, ……, $\frac{V_a}{V_b} = \frac{R_a}{R_b}$.

Components connected in parallel



1) $V_{AB} = V_1 = V_2 = V_3 = ...$
2) $I = I_A = I_1 + I_2 + I_3 + ... = I_B$
3) $R_T = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...}$ *remains constant for ohmic resistors. Also* $R_T = \frac{V_{AB}}{I}$
4) $\frac{I_1}{I_2} = \frac{R_2}{R_1}$, $\frac{I_2}{I_3} = \frac{R_3}{R_2}$, ……, $\frac{I_a}{I_b} = \frac{R_b}{R_a}$

Example 1



(a) Find the potential difference and current across each component.
(b) Calculate the total resistance of the circuit in two ways.

(a) $V_1 = I_1 R_1 = (2.0\times10^3)(0.90\times10^{-3}) = 1.8\text{ V}$

$V_2 = V_1 = 1.8\text{ V}$. $I_2 = \frac{V_2}{R_2} = \frac{1.8}{3.0\times10^3} = 0.60\times10^{-3}\text{ A} = 0.60\text{ mA}$

$I_3 = I_1 + I_2 = 0.90 + 0.60 = 1.5\text{ mA}$

$V_3 = I_3 R_3 = (1.5\times10^{-3})(1.8\times10^3) = 2.7\text{ V}$

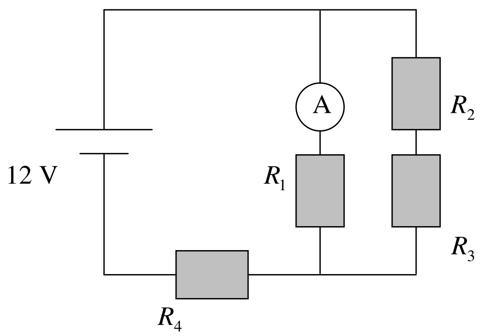
$I_4 = I_3 = 1.5\text{ mA}$. $V_4 = I_4 R_4 = (1.5\times10^{-3})(1.0\times10^3) = 1.5\text{ V}$

$I_{battery} = I_4 = 1.5\text{ mA}$. $V_{battery} = 1.8 + 2.7 + 1.5 = 6.0\text{ V}$

(b) $R_T = \frac{V_{battery}}{I_{battery}} = \frac{6.0}{1.5\times10^{-3}} = 4.0\text{ k}\Omega$

Alternatively, $R_T = \frac{1}{\frac{1}{2.0} + \frac{1}{3.0}} + 1.8 + 1.0 = 4.0\text{ k}\Omega$

Example 2 The resistors in the following circuit are identical and each has a resistance of R Ω, i.e. $R_1 = R_2 = R_3 = R_4 = R$. The ammeter reading is 5.0 mA. Find R.



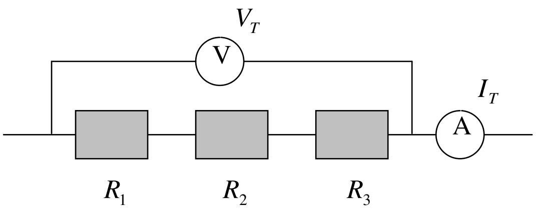
$$\frac{I_2}{I_1} = \frac{R}{R+R} = \frac{1}{2}. \ I_2 = \frac{1}{2} I_1 = \frac{1}{2} \times 5.0 = 2.5 \text{ mA}$$

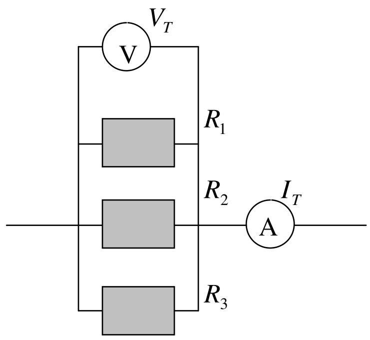
$$I_4 = I_{battery} = 5.0 + 2.5 = 7.5 \text{ mA}$$

$$R_T = \frac{V_{battery}}{I_{battery}} = \frac{12}{7.5 \times 10^{-3}} = 1.6 \text{ k}\Omega$$

$$\frac{1}{\frac{1}{R} + \frac{1}{2R}} + R = 1.6, \ \frac{5R}{3} = 1.6, \ R = 0.96 \text{ k}\Omega = 960 \ \Omega$$

Power in series and parallel connections





For both types of circuits, $P_T = P_1 + P_2 + P_3 + \ldots$, *where*

$$P_T = V_T I_T = \frac{V_T^2}{R_T} = I_T^2 R_T$$

$$P_1 = V_1 I_1 = \frac{V_1^2}{R_1} = I_1^2 R_1$$

$$P_2 = V_2 I_2 = \frac{V_2^2}{R_2} = I_2^2 R_2$$

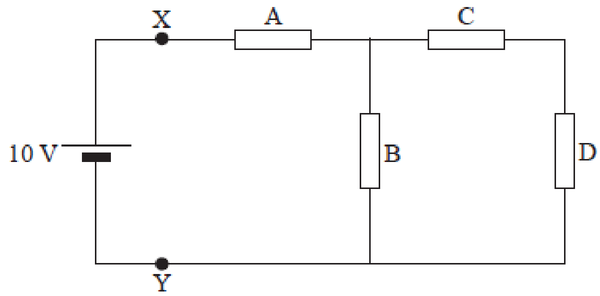
$$P_3 = V_3 I_3 = \frac{V_3^2}{R_3} = I_3^2 R_3$$

The total power consumption in a parallel or series connection of components is the sum of the individual power of the components in the connection.

The power of a component in a parallel connection is the same power as when it is alone. Hence the total power of the parallel circuit is higher than that of a single component alone.

The power of a component in a series connection is less than the power of the component when it is alone, and the total power of the series circuit is less than that of a single component alone.

Example 3 (2011 VCAA Exam 1)
Four 2.0 ohm resistors (A, B, C and D) are connected as in Figure



(a) Show that the total resistance of the circuit between X and Y is 3.3 ohm.
(b) A 10 V battery is now connected across XY as shown
What is the current through resistor B?
(c) What is the voltage drop (potential difference) across resistor A?
(d) What is the power dissipated in resistor D?

(a) $R_{total} = 2.0 + \frac{1}{\frac{1}{2.0+2.0} + \frac{1}{2.0}} = 3.3 \ \Omega$

(b) $I_{battery} = \frac{V_{XY}}{R_{total}} = \frac{10}{\frac{10}{3}} = 3.0 \text{ A}$, .: $I_B + I_C = 3.0 \text{ A}$

$V_B = V_C + V_D$

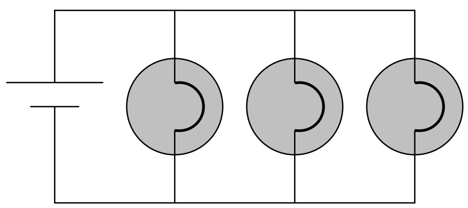
.: $I_B R_B = I_C R_C + I_D R_D = I_C (R_C + R_D)$, since $I_C = I_D$, .: $I_B = 2 I_C$

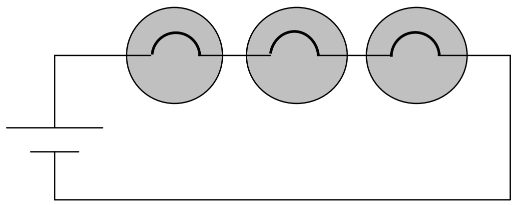
Hence $I_B = 2.0 \text{ A}$

(c) $V_A = I_A R_A = 3 \times 2 = 6.0 \text{ V}$

(d) $P_D = I_D^2 R_D = 1 \times 2 = 2.0 \text{ W}$

Example 4 Three identical light globes (3W 12V) are connected in parallel and then in series. In each case the power is supplied by a 12V battery. The globes are assumed to be ohmic conductors.
(a) Calculate the resistance of each globe.
(b) Find the power of each globe and the total power dissipated in the parallel circuit.
(c) Calculate the power of each globe and the total power dissipated in the series circuit.
(d) Calculate the current through the battery in each circuit.





(a) $R = \frac{V^2}{P} = \frac{12^2}{3} = 48 \ \Omega$

(b) $P = 3 \text{ W}$. $P_T = 3 + 3 + 3 = 9 \text{ W}$

(c) $P = \frac{V^2}{R} = \frac{4^2}{48} = \frac{1}{3} = 0.33 \text{ W}$, $P_T = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \text{W}$

(d) $I_{battery} = \frac{P_T}{V_{battery}} = \frac{9}{12} = 0.75$ A for the parallel circuit

$I_{battery} = \frac{1}{12} = 0.083$ A for the series circuit

Exercise: Next page

Q1 Two ohmic conductors R_1 and R_2 are in series. $R_1 = 100\Omega$, $R_2 = 150\Omega$ and $V_1 + V_2 = 8$ V. Find V_1.

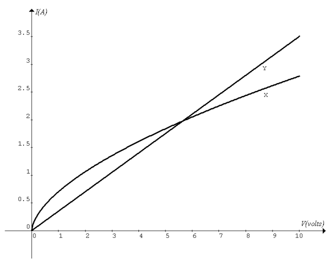
Q2 A car battery provides 30 J of energy to 2.5 C of charge to start a car. What is the emf in volts of the battery?

Q3 A battery provides 1.5 J of potential energy to each coulomb of charge which leaves it over a 5.0 s interval. It supplies a current of 320 mA. How much potential energy is provided in the 5.0 s interval?

Q4 A battery of emf 3.0 V supplies a current of 2.0 μ A in a digital watch. What is the total amount of potential energy provided by the battery to the current in 2.0 minutes?

Q5 A current of 1.5 A flows through a resistance wire when it is connected to points X and Y in a circuit. X has a potential of 8.0 V and Y has a potential of 2.0 V. Calculate the resistance of the wire.

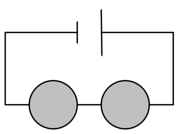
Q6 The graphs show the I-V characteristics of two conductors X and Y. Which one has a higher resistance (a) at 1 V potential difference? (b) when the current is 2.5 A?



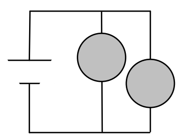
Q7 Refer to the I-V characteristics of two conductors X and Y in Q6. (a) At what potential difference do they have the same resistance? (b) What is this resistance?

Q8 A heating resistance wire is connected between the terminals of a 12 V battery. The power dissipated in the resistor is 20 W. The same wire is then connected between the terminals of a 6 V battery. What power is dissipated in this case?

Q9 Two identical lights, each giving 6 W of power, are connected as shown. The current in the battery is 2.0 A.
(a) What is the current in the first light? (b) What is the potential difference between the terminals of each light?
(c) What is the voltage of the battery? (d) How much energy is drained from the battery in a minute?



Q10 Two identical lights, each giving 6 W of power, are connected as shown. The current in the battery is 2.0 A.
(a) What is the current in the first light? (b) What is the potential difference between the terminals of each light?
(c) What is the voltage of the battery? (d) How much energy is drained from the battery in a minute? (e) Are the lights in Q9 identical to the lights in this question?



Answers: 1. 3.2 V 2. 12 V 3. 2.4 J 4. 7.2×10^{-4} J 5. 4.0 Ω 6a. Y 6b. X 7a. 5.5 V 7b. 2.8 Ω 8. 5 W
9a. 2.0 A 9b. 3.0 V 9c. 6.0 V 9d. 720 J 10a. 1.0 A 10b. 6.0 V 10c. 6.0 V 10d. 720 J 10e. No, lights in Q9 have lower resistance.