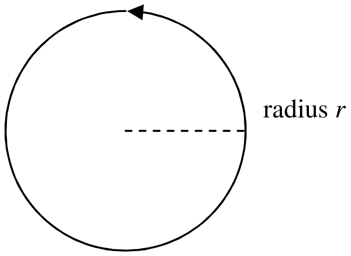
Free physics self-help revision

Uniform circular motion

*When an object travels at **constant speed** in a circle, its motion is described as **uniform** circular motion.*



The time for it to complete one revolution is the period T of the motion. The number of revolutions completed in a unit time is the frequency f.

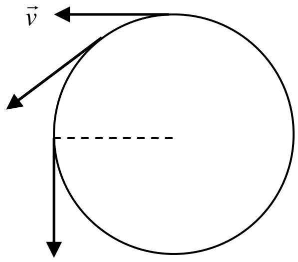
$$f = \frac{1}{T}$$

The speed v of the object can be calculated by

$$speed = \frac{distance\ travelled}{time\ taken}.$$

Other relationships are: $v = \frac{2\pi r}{T}$, $v = 2\pi r f$

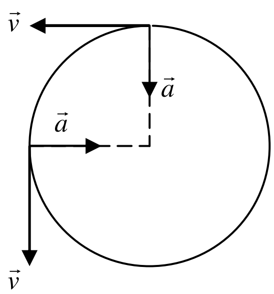
Although the speed is constant, the velocity is not because the direction of motion changes continuously as the object moves in a circle. The velocity vector $\vec{v}$ *is tangential to the path.*



$\therefore$ *the object has an acceleration* $\vec{a}$. *The magnitude of* $\vec{a}$ *is given by*

$$a = \frac{v^2}{r}$$

The direction of $\vec{a}$ *is always towards the centre of the circle, i.e. inwards along the radius of the circular path. Thus the acceleration* $\vec{a}$ *is sometimes called centripetal acceleration. Since v and r are constant,* $\therefore$ *a is constant but* $\vec{a}$ *is not because its direction changes continuously as the object changes its position in the circle.*



$\vec{v}$ *and* $\vec{a}$ *are perpendicular to each other.*

If the period T (or frequency f) and r are known, then

$$a = \frac{4\pi^2 r}{T^2} \text{ or } a = 4\pi^2 r f^2.$$

Example 1 A racing car completes 5 laps of a round race track of radius 250 m in 5.35 min.
(a) Determine the period in hours.
(b) Determine the frequency in laps per hour.
(c) Determine the average speed in km h^{-1}.
(d) What is the average velocity in km h^{-1}?

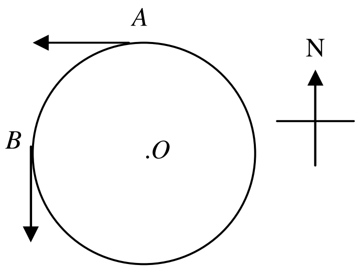
(a) $T = \frac{5.35}{5} = 1.07 \text{ min} = 0.018 \text{ h}$

(b) $f = \frac{1}{T} = 56$ laps per hour

(c) $v_{av} = \frac{2\pi r}{T} = 88.1 \text{ km h}^{-1}$

(d) $\vec{v}_{av} = \frac{s}{\Delta t} = 0$

Example 2 A cyclist travels around a roundabout of radius 6.2 m at 3.1 ms^{-1}.



(a) Determine her accelerations at *A* and *B*.
(b) Determine her positions at *A* and *B* relative to the centre *O* of the roundabout.
(c) Calculate her average velocity from *A* to *B*.
(d) Determine her velocities at *A* and *B*.
(e) Calculate her average acceleration from *A* to *B*.

(a) $a = \frac{v^2}{r} = 1.6 \text{ m s}^{-2}$

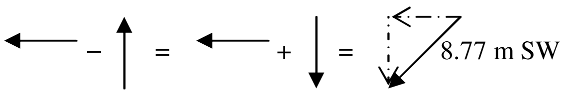
1.6 m s^{-2} south at *A*, 1.6 m s^{-2} east at *B*

(b) 6.2 m north of *O* at *A*, 6.2 m west of *O* at *B*

(c) Time for one round $= T = \frac{2\pi r}{v} = 12.57$ s

Time to go from *A* to $B = \frac{T}{4} = 3.14$ s

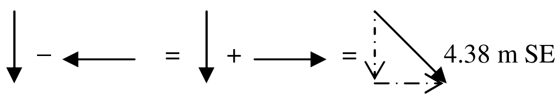
From *A* to *B*, displacement $s = 6.2$ m W -6.2 m N



$\vec{v}_{av} = \frac{s}{\Delta t} = 2.8 \text{ m s}^{-1}$ SW

(d) $\vec{v}_A = 3.1 \text{ m s}^{-1}$ W, $\vec{v}_B = 3.1 \text{ m s}^{-1}$ S

(e) From *A* to *B*, $\Delta\vec{v} = \vec{v}_B - \vec{v}_A$



$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = 1.4 \text{ m s}^{-2}$ SE

Example 3 A 1500 kg car travels at 15 m s^{-1} around a flat horizontal bend of radius 50 m.

(a) Determine the total friction (between the tyres and the road surface) which helps the car to make the turn.
(b) If the maximum possible friction is 0.6 times the normal reaction of the road on the car, what is the maximum speed the car can have before it skids out of control?

(a) $a = \frac{v^2}{r} = \frac{15^2}{50} = 4.5$ m s^{-2}

$F_{net} = ma$, $F_{net} = F_f = 1500 \times 4.5 = 6.75 \times 10^4$ N

(b) Max acceleration:

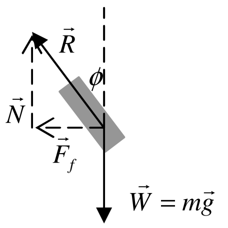
$a_{max} = \frac{F_{f,max}}{m} = \frac{0.6 \times 1500 \times 9.8}{1500} = 5.88$ m s^{-2}

$a_{max} = \frac{v_{max}^2}{r}$, $v_{max} = \sqrt{a_{max} r} = 17$ m s^{-1}

Example 4 Refer to previous discussion of a motorcyclist moving around a bend. The total mass of the rider and the motorcycle is 850 kg, and the motorcycle makes an angle of 20° with the vertical. The radius of the bend is 50 m.

(a) Determine the normal reaction N of the road surface on the cycle.
(b) Calculate the friction F_f between the tyres and the road.
(c) Determine the acceleration a of the motorcycle.
(d) Calculate the speed v of the motorcycle.
(e) What is the speed if another person who is 10 kg heavier rides the same motorcycle under the same conditions?
(f) Show the relationship between angle ϕ with the vertical, speed v of the motorcycle and radius r of the bend.

(a)



$N = W = mg = 850 \times 9.8 = 8330$ N

(b) $\frac{F_f}{N} = \tan\phi$, $F_f = N\tan\phi = 8330 \tan 20° = 3.0 \times 10^3$ N

(c) $a = \frac{F_{net}}{m} = \frac{F_f}{m} = 3.6$ m s^{-2}

(d) $a = \frac{v^2}{r}$, $v = \sqrt{ar} = 13$ m s^{-1}

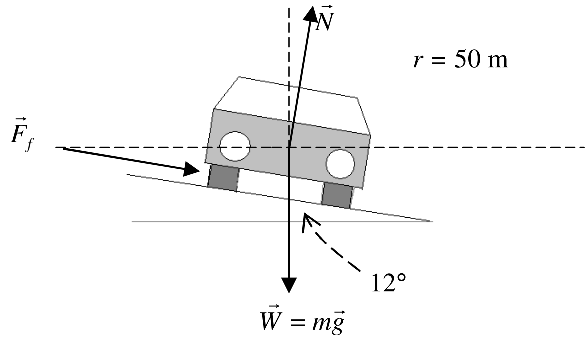
(e) Same speed because speed around the bend is independent of mass. $N = W = mg$, $F_f = N\tan\phi = mg\tan\phi$,

$a = \frac{F_{net}}{m} = \frac{F_f}{m} = g\tan\phi$, $\therefore v = \sqrt{ar} = \sqrt{gr\tan\phi}$.

(f) $\tan\phi = \frac{v^2}{gr}$, $\phi = \tan^{-1}\left(\frac{v^2}{gr}\right)$, provided $F_{f,max} \geq mg\tan\phi$, i.e. as long as the friction between the tyres and the road is less than the maximum possible friction, the leaning angle ϕ increases with increasing speed v, and with smaller radius r.

Example 5 A 1500-kg car enters a turn whose radius is 50 m. The road is banked at an angle of 12°. The maximum friction force between the tyres and the road surface is 0.6 times the normal reaction force of the road on the car.

(a) Calculate the force of friction F_f on the tyres when the car travels at a constant speed of 15 m s^{-1}.
(b) Find the maximum speed v_{max} of the car before it skids out of control.
(c) Find the speed that no friction is required to make the turn. Note: This speed is called the *design speed* for the turn.
(d) State the benefits of a banked road to motorists making a turn.



(a) There are 3 forces on the car: $\vec{N}$, $\vec{W}$ and $\vec{F}_f$.
Resolve the forces into vertical and *horizontal* components because the car is in *horizontal* uniform circular motion.
Vertically: $F_{net} = 0$, $N\cos 12° - F_f \sin 12° - 1500 \times 9.8 = 0$.

$N - F_f \tan 12° = \frac{1500 \times 9.8}{\cos 12°}$, $N - 0.2126 F_f = 15028$ ……(1)

Horizontally: $F_{net} = ma$, $F_{net} = \frac{mv^2}{r}$,

$F_f \cos 12° + N \sin 12° = \frac{1500 \times 15^2}{50}$.

$\frac{F_f}{\tan 12°} + N = \frac{1500 \times 15^2}{50 \sin 12°}$, $4.7046 F_f + N = 32466$ ……(2)

(2) − (1), $4.9172 F_f = 17437$, $F_f = 3.5 \times 10^3$ N.
Note: Compare with Example 3(a).

(b) Maximum friction force $F_{f,max} = 0.6N$.
Vertically: $F_{net} = 0$, $N\cos 12° - F_f \sin 12° - 1500 \times 9.8 = 0$,
$N\cos 12° - 0.6N \sin 12° - 1500 \times 9.8 = 0$, $N = 17225$.

Horizontally: $F_f \cos 12° + N\sin 12° = \frac{1500 \times v^2}{50}$,

$0.6N\cos 12° + N\sin 12° = \frac{1500 \times v^2}{50}$, $\therefore v = 21.4$ m s^{-1} is the maximum speed. Note: Compare with Example 3(b).

(c) $F_f = 0$
Vertically: $F_{net} = 0$, $N\cos 12° = 1500 \times 9.8$ ……(1)

Horizontally: $N \sin 12° = \frac{1500 \times v^2}{50}$ ……(2)

(2)/(1) : $\tan 12° = \frac{v^2}{9.8 \times 50}$, $v = 10.2$ m s^{-1}.

Note: $v = \sqrt{gr\tan\theta}$ gives the design speed.

(d) *A banked road (i) creates an extra horizontal component of normal reaction force and thus the centripetal force required to make a turn is less reliant on friction force as its source;*
(ii) increases the normal reaction force on the car during the turn and thus increases the maximum friction force between the tyres and the road.
Hence the maximum allowable speed is increased.

Questions: Next page

Q1a A 1200-kg car travels at 32 ms^{-1} for 3 minutes around a circular track of radius 450 m. Find the number of complete laps finished by the car.

Q1b Determine the acceleration of the car while it is in motion.

Q1c Find the magnitude of the average velocity of the car in one half of a lap.

Q1d Find the magnitude of the average acceleration of the car in one half of a lap.

Q1e Determine the centripetal force of friction on the tyres to keep the car moving around the circular track at 32 ms^{-1}.

Q1f The car will skid out of control if the centripetal force of friction on the tyres reaches 3200 N. Determine the speed range for safe driving.

Q2a A motorcyclist moves around a bend. The total mass of the rider and the motorcycle is 650 kg, and the motorcycle makes an angle of 15° with the vertical. The radius of the bend is 60 m. Determine the normal reaction *N* of the road surface on the cycle.

Q2b Calculate the friction F_f between the tyres and the road.

Q2c Determine the acceleration *a* of the motorcycle.

Q2d Calculate the speed *v* of the motorcycle.

Q3a A 1000-kg car enters a curved road whose radius is 60 m. The road is banked at an angle of 10°. The maximum friction force between the tyres and the road surface is 0.5 times the normal reaction force of the road on the car. Calculate the design speed for the curved road.

Q3b Does the design speed depend on the total mass of the car, and the weather conditions?

Numerical, algebraic and worded answers:

1a. 2 laps 1b. 2.3 m s^{-1} 1c. 20.4 m s^{-1} 1d. 1.4 m s^{-2} 1e. 2.7×10^3 N 1f. less than 34.6 m s^{-1} 2a. 6370 N 2b. 1707 N 2c. 2.6 m s^{-2} 2d. 12.6 m s^{-1}
3a. 10.2 m s^{-1} 3b. No, it depends on the radius of curvature and the banked angle only.