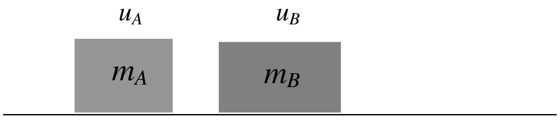
Free physics self-help revision

Conservation of momentum and Newton's third law

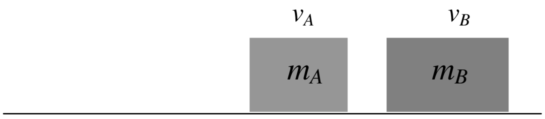
Consider a system of two objects A and B sliding along a smooth surface in a straight line, approaching and colliding with each other head on. The system can be considered ***isolated*** *because the net external force on it is zero. Friction is negligible and the force of gravity is balanced by the normal reaction force. Within the system during collision there are two interacting forces, F_A on object A and F_B on object B.*

Before collision

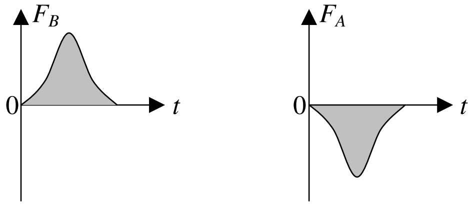


During collision

After collision



During collision, according to Newton's third law, $F_A = -F_B$.



The impulse (area 'under' the force-time graph) given to A by B is equal but opposite to that given to B by A.

Therefore, according to Newton's second law, the change in momentum of A is equal but opposite to that of B.

$$m_B v_B - m_B u_B = -(\, m_A v_A - m_A u_A \,)$$

$$\therefore\ m_A v_A + m_B v_B = m_A u_A + m_B u_B$$

i.e. the total momentum after collision is equal to the total momentum before collision. Even during collision, total momentum is also conserved. This is known as the Law of conservation of momentum. The total momentum remains constant for an isolated system.

In real life situations, e.g. a two-car head on collision, only the total momentum ***immediately*** *before and after the collision are considered otherwise the effects of friction are too great to be ignored and the system can no longer be treated as isolated.*

Example 1 Due to signal malfunction, a train of mass M kg travelling at 30 km h^{-1} collides with another of mass 2.5M kg travelling in the opposite direction at 35 km h^{-1}. The two trains lock together after the collision.
(a) Determine the velocity of the locked trains immediately after the collision.
(b) In terms of M, calculate the total kinetic energy of the two trains immediately before the collision.
(c) Compare the answer in (b) with the total kinetic energy immediately after the collision. Discuss the difference.

(a) It is not necessary to change km h^{-1} to m s^{-1} in this part.
Total momentum just after collision = total momentum just before collision

$(M + 2.5M)\vec{V} = M \times {}^{+}30 + 2.5M \times {}^{-}35$

$3.5\vec{V} = {}^{-}57.5$, $\therefore \vec{V} = {}^{-}16.4$ km h^{-1}

(b) Before collision:
30 km h^{-1} = 8.33 m s^{-1}, 35 km h^{-1} = 9.72 m s^{-1}

Total kinetic energy $= \frac{1}{2} \times M \times 8.33^2 + \frac{1}{2} \times 2.5M \times 9.72^2$

$\approx 153M$ J.

(c) After collision: $\vec{V} = {}^{-}16.4$ km h^{-1} = ${}^{-}4.56$ m s^{-1}

Total kinetic energy $= \frac{1}{2} \times (M + 2.5M) \times 4.56^2 \approx 36M$ J

Some of the kinetic energy before collision is transformed to heat, sound and strain energies after collision.
The type of collision discussed in the last example is described as an ***inelastic collision****. In an inelastic collision, the total kinetic energy after collision is different from (note: not always less than, e.g. when a rifle is fired) the total kinetic energy before collision. If the total kinetic energy remains constant before and after collision, the collision is called* ***elastic****. In both elastic and inelastic collisions the total momentum is conserved.*

Example 2 A 3 kg mass A moving at 2 m s^{-1} collides with a 2 kg mass B moving in the same direction at 1 m s^{-1} on a frictionless horizontal surface. After collision the two masses continue to move in the same straight line and the speed of mass A is 1.3 m s^{-1}. Determine whether the collision is elastic or inelastic.

Conservation of momentum: $3\vec{v}_A + 2 \times {}^{+}1.3 = 3 \times {}^{+}2 + 2 \times {}^{+}1$

$\vec{v}_A = {}^{+}1.8$ m s^{-1}
Before the collision:

Total kinetic energy $= \frac{1}{2} \times 3 \times 2^2 + \frac{1}{2} \times 2 \times 1^2 = 7$ J

After the collision:

Total kinetic energy $= \frac{1}{2} \times 3 \times 1.8^2 + \frac{1}{2} \times 2 \times 1.3^2 \approx 6.6$ J

.: the collision is inelastic.

Example 3 Two identical train carriages A and B travel on the same tracks in the same direction. Carriage A travels at 2 m s^{-1} and rolls into carriage B travelling at 1 m s^{-1}. Assume that it is an elastic collision. Calculate the velocities of A and B immediately after the collision.

Let v_A and v_B be the velocities of A and B after collision respectively.
Conservation of momentum:
$mv_A + mv_B = m \times 2 + m \times 1$, $\therefore v_A + v_B = 3$ ……(1)
Elastic collision:

$\frac{1}{2} m{v_A}^2 + \frac{1}{2} m{v_B}^2 = \frac{1}{2} m \times 2^2 + \frac{1}{2} m \times 1^2$

$\therefore {v_A}^2 + {v_B}^2 = 5$ ……(2)

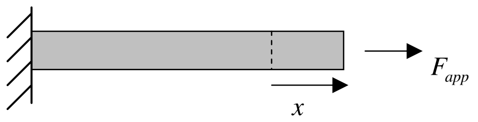
Square (1), ${v_A}^2 + 2v_A v_B + {v_B}^2 = 9$ ……(3)

(3) – (2), $v_A v_B = 2$, $v_B = \frac{2}{v_A}$ ……(4)

Substitute (4) in (1), $v_A + \frac{2}{v_A} = 3$, $\therefore {v_A}^2 - 3v_A + 2 = 0$

Hence $v_A = 1$ m s^{-1} and $v_B = 2$ m s^{-1}

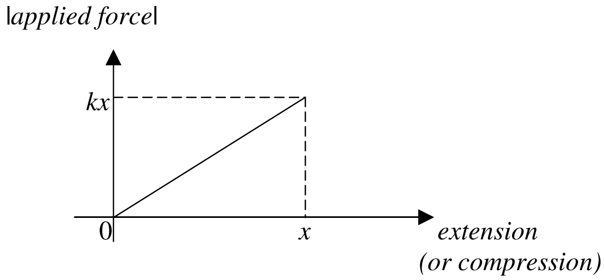
Hooke's law and elastic potential energy



When a material (or spring) is compressed or stretched, the applied force F_{app} is directly proportional to the amount x compressed or stretched provided it is not overdone. This is known as Hooke's law.

$F_{app} \propto x$, $\therefore F_{app} = kx$ *where k is called the force constant of the material and has the unit N m^{-1} if F_{app} is measured in N and x is in m.*

Hooke's law can also be expressed in terms of the force F exerted by a material (or spring), $F = -kx$.



The value of k is given by the gradient of the F-x graph.

The amount of work done (J) by the applied force or the amount of elastic strain energy (J) stored in the material is given by the area under the F-x graph.

$$W = E_p = \frac{1}{2}F_{app}x = \frac{1}{2}kx^2$$

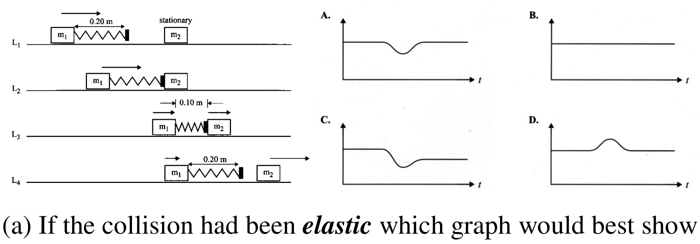
When the amount of compression or extension changes from x_a to x_b, the additional work done or the increase in elastic strain energy is $W = \Delta E_p = \frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2$

Example 4 A 2.0 N force is required to stretch a spring by 2.5 cm.
(a) Find the extension of the spring when it exerts a force of 3.5 N on each of your hands.
(b) Calculate the elastic potential energy of the spring when the applied force is 3.5 N.

(a) $\frac{x_b}{F_{app,b}} = \frac{x_a}{F_{app,a}}$, $\frac{x_b}{3.5} = \frac{2.5}{2.0}$, $x_b \approx 4.4$ cm (4.375 cm)

(b) $E_p = \frac{1}{2} \times 3.5 \times 4.375 \approx 7.7$ J

Example 5 (*2010 VCAA Exam 1*) Physics students are conducting a collision experiment using two trolleys m_1 of mass 0.40 kg and m_2 of mass 0.20 kg on a frictionless surface.



(a) If the collision had been ***elastic*** which graph would best show how (i) the total kinetic energy (ii) the total momentum of the system varies with time before, during and after the collision?
(b) If the collision had been ***inelastic*** which graph would best show how (i) the total kinetic energy (ii) the total momentum of the system varies with time before, during and after the collision?
Answers: ai A, aii B, bi C, bii B

Example 6 The extension of a rubber band increases by 1.0 cm when the applied force increases by 1.0 N.
(a) Find the force constant of the rubber band.
(b) Find the initial extension when the extra work required to extend it 1.0 cm further is 0.020 J.

(a) $k = \frac{\Delta F_{app}}{\Delta x} = \frac{1.0}{1.0 \times 10^{-2}} = 100$ N m^{-1}

(b) $W = \frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2$, $0.020 = \frac{1}{2} \times 100\left((x_a + 0.010)^2 - x_a^2\right)$

$0.00040 = 0.020x_a + 0.00010$, $x_a = 0.015$ m

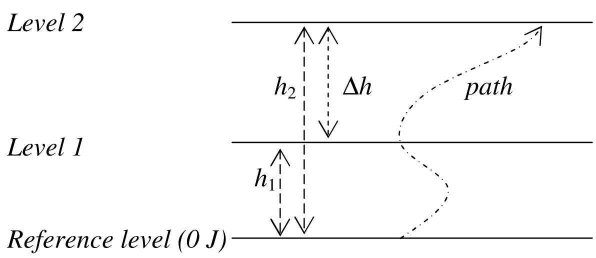
Gravitational potential energy

To measure gravitational potential energy at (near) the surface of the earth, a level (usually the lowest, but not necessarily) is chosen as the reference level and assigned a value of 0 J. Above this reference level the gravitational potential energy is a positive value, below it is negative.

Consider two levels above the reference level. Level 1 is h_1 metres above the reference level and Level 2 is h_2 metres above the reference level. To lift a m kg object at constant speed, a force equals to the weight of the object, i.e. mg newtons, is required. The work done in lifting the object from the reference level to Level 1 is mgh_1 J, and from the reference level to Level 2 is mgh_2 J. The object's gravitational potential energy at each level is mgh_1 J and mgh_2 J respectively. Gravitational potential energy depends on the vertical distance only and not the path of motion from one level to another.

The difference (or change) in gravitational potential energy when the object is raised from Level 1 to Level 2 is

$$\Delta E_p = mgh_2 - mgh_1 = mg(h_2 - h_1) = mg\Delta h$$



Example 7 (*2010 VCAA Exam 1*) An ideal spring has a 2.0 kg mass attached to it. The spring-mass system is initially held so that the spring is not extended. The mass is gently lowered and the spring stretches until the spring-mass system is finally at rest. The spring has extended by 0.40 m. (a) Find the value of the spring constant. (b) Find the difference in magnitude between the initial and final total energy of the spring-mass system.

(a) When the mass is at rest, it exerts a force of $F_{app} = W = mg = 2.0 \times 10 = 20$ N on the spring and stretches it by 0.40 m, $\therefore$ $k = \frac{F_{app}}{x} = \frac{20}{0.40} = 50$ N m^{-1}

(b) Let the final position (the lowest point) of the mass be the reference level and the potential energy of the mass be 0J.
Initially: *Total energy* $= mgh = 2.0 \times 10 \times 0.40 = 8.0$ J

Finally: *Total energy* $= \frac{1}{2}kx^2 = \frac{1}{2} \times 50 \times 0.40^2 = 4.0$ J

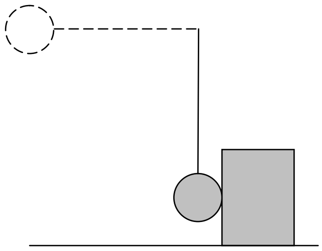
Difference $= 8.0 - 4.0 = 4.0$ J

Questions: Next page

Q1 On an icy road a 3 tonne truck travelling at 54 km h^{-1} east collides head on with a 1 tonne car travelling at 72 km h^{-1} west. (a) What is the total momentum of the two vehicles immediately after collision? (b) What is the *net* impulse on the two vehicles immediately after collision?

Q2 Refer to Q1. The drivers of the two vehicles have the same mass. Both are properly restrained by seatbelts. Which driver experiences stronger force from the seatbelt? Give an explanation to your answer.

Q3 A steel ball (0.500 kg) is fastened to a cord of 70.0 cm long and fixed at the far end, and is released when the cord is horizontal. At the bottom of its path, the ball strikes a steel block (2.50 kg) initially at rest on a frictionless surface. The block moves away with a speed of 1.15 m s^{-1}. Find the velocity of the ball immediately after collision.

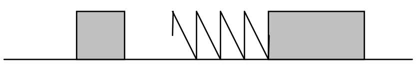


Q4 Refer to Q3. If the collision is elastic. Find (a) the speed of the ball (b) the speed of the block, both immediately after collision.

Q5 A 5.20 g bullet moving at 672 m s^{-1} strikes a 0.700 kg wooden block at rest on a frictionless surface. The bullet emerges with a speed of 428 m s^{-1}. Find the resulting speed of the block.

Q6 When a mass of m kg is attached to a vertically hung spring of spring constant k, the length of the spring is 0.8 m. When a mass of 2m kg is attached to the spring, the length becomes 0.9 m. Calculate the value of the ratio $\frac{k}{m}$.

Q7 A block $m_1 = 2.0$ kg slides along a frictionless table at 10 m s^{-1}. Directly in front of it and moving in the same direction, is a block $m_2 = 5.0$ kg at 3.0 m s^{-1}. A massless spring ($k = 1120$ N m^{-1}) is attached to the near side of m_2. When the blocks collide, what is the maximum compression of the spring? m_1 m_2



Q8 A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above it. (a) What is the change in gravitational potential energy of the marble during its ascent? (b) What is the spring constant?

Numerical, algebraic and worded answers: 1a. 25000 kg m s^{-1} 1b. 0 N s 2. Same force in magnitude on each vehicle. The car has a smaller mass .; larger acceleration .: larger force on the driver from the seatbelt. 3. 2.59 m s^{-1} backward 4a. 2.68 m s^{-1} backward 4b. 1.17 m s^{-1} forward 5. 1.81 m s^{-1} forward 6. 100 7. 0.25 m 8a. 1 J 8b. 312.5 N m^{-1}