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The law of conservation of energy

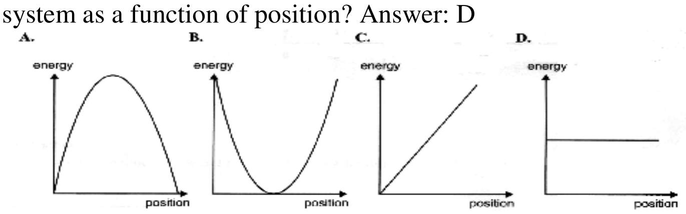
Energy changes from one form to another and can be transferred from one object to another during interaction. Work is done during transformation or transfer of energy.
For example, in a car collision the head of the driver presses against the airbag. Work is done by the head to compress the airbag. In doing so the kinetic energy of the head changes to strain energy E_s in deforming the airbag and some heat is also generated.

$$E_k \xrightarrow{Work} E_s + heat$$

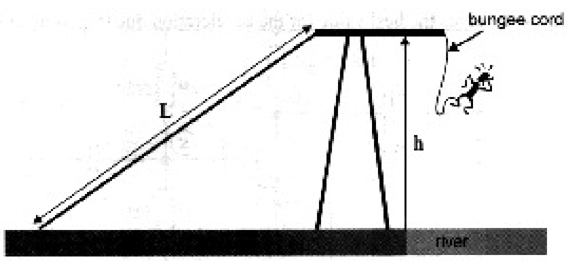
To a good approximation the total amount of energy (E_k + E_s + heat) at any time during the collision of the head with the airbag is constant. This example is an approximation because the system of objects (the head and the airbag) is not isolated for obvious reasons.

For an isolated system, the total amount of energy is constant. This is known as the ***law of conservation of energy****.*

Example 1 (2007 VCAA Exam 1) A mass is attached to a spring hanging vertically. Which one of the graphs could best represent the variation of the total energy of the oscillating mass-spring system as a function of position? Answer: D



Example 2 (2006 VCAA Exam 1) Sam (70 kg) is bungee jumping from a platform at the top of a tower. He reaches the top of the tower by being towed up a slide of length L. The friction between Sam and the slide provides a constant force of 300 N that opposes the motion. The total work done in dragging Sam up the slide to the top of the tower is 22720 J. At the top of the tower Sam's potential energy was greater by 13720 J than it was on the ground.



(a) Show that $L = 30$ m. (b) Find the height h (m) of the tower. (c) The natural length of the bungee cord is 10 m. Sam stops falling and first comes to rest momentarily when the length of the cord is 18 m. Find the spring constant of the bungee cord.

(a) *The total work done in dragging Sam up the slide*
$= 300L + 13720 = 22720$, .: $L = 30$ m

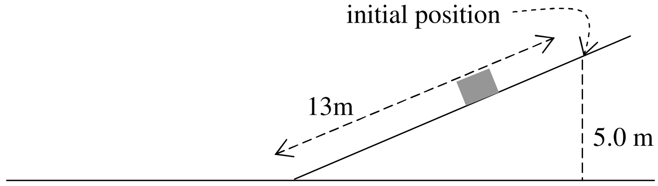
(b) *Increase in potential energy* $= mgh$
$13720 = 70 \times 10 \times h$, .: $h = 19.6$ m

(c) $|\Delta E_g| = |\Delta E_e|$, $mgh = \frac{1}{2}kx^2$, $70 \times 10 \times 18 = \frac{1}{2} \times k \times (18 - 10)^2$

$k = 3.9 \times 10^2$ N m^{-1}

Example 3 Due to handbrake failure a parked 1000 kg mini-van rolled down an incline onto a horizontal straight stretch of road. It came to a stop after travelling 37 m along the horizontal section. See the diagram below for further information.

(a) Determine the average resistive force opposing its motion.
(b) Assume the same average resistive force on the slope and on the horizontal section. Find the speed of the van when it entered the horizontal section.



(a) When the van came to a stop the amount of heat generated by the resistive force (air resistance and rolling frictions)
$= F_{resis,av} \times d$, where d is the total distance travelled.
At the initial position the total energy $= mgh$.
$\therefore F_{resis,av} \times d = mgh$, $F_{resis,av} \times (13 + 37) = 1000(9.8)(5.0)$
$F_{resis,av} = 980$ N.

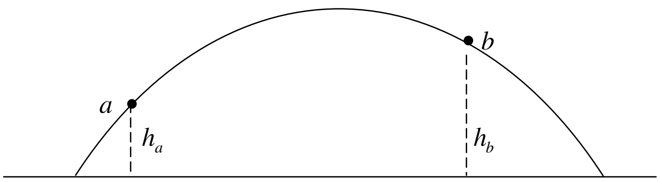
(b) $mgh_a + \frac{1}{2}mv_a^2 + heat = mgh_b + \frac{1}{2}mv_b^2$

$0 + \frac{1}{2}(1000)v_a{}^2 + 980 \times 13 = 1000(9.8)(5.0) + 0$, $v_a \approx 8.5$ m s^{-1}

Conservation of energy and projectile motion

The system consists of the projectile and the earth. It can be considered as isolated, and there are two types of energy involved. They are the kinetic energies of the projectile and the earth, and the gravitational potential energy between them.

The mass of the earth is very much greater than the mass of the projectile. This results in practically zero speed for the earth relative to the centre of mass of the system. Therefore when one applies the law of conservation of energy to projectile motions, it is simpler to take the total amount of energy as the sum of the gravitational potential energy and the kinetic energy of the projectile, and it is constant at different stages of the motion.



$$mgh_a + \frac{1}{2}mv_a^2 = mgh_b + \frac{1}{2}mv_b^2 \qquad (1)$$

or

$$\Delta E_k + \Delta E_p = 0 \qquad (2)$$

Equation (2) suggests that when kinetic energy increases (decreases), potential energy decreases (increases) by the same amount and the changes always add to zero.

If air resistance is taken into account, heat should be included in both equations.

Example 4 Due to excessive speed a car failed to make a turn and crashed through the railings at 72 km h^{-1} (20 m s^{-1}) on top of a 10 m cliff.

(a) At what speed did it hit the water?
(b) Would it make any difference whether the road inclined upwards or downwards at the crash site?

(a) Take 0 J as the gravitational potential energy at the surface of water.

Use $mgh_a + \frac{1}{2}mv_a^2 = mgh_b + \frac{1}{2}mv_b^2$.

$0 + \frac{1}{2}mv_a^{\ 2} = m(9.8)(10) + \frac{1}{2}m(20^2)$

$v_a = 24$ m s^{-1} was the speed of the car when it hit the water.

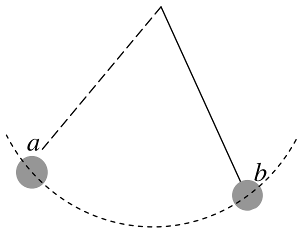
(b) The relationship $mgh_a + \frac{1}{2}mv_a^2 = mgh_b + \frac{1}{2}mv_b^2$ does not depend on the inclination of the road, $\therefore$ same speed.

Conservation of energy and vertical circular motion

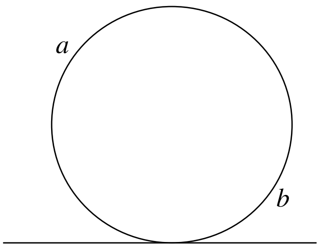
Consider the system consisting of the earth and an object moving in a vertical circle. Similar to the projectile motion in the previous section, the system can be simplified to that of an object moving in a vertical circle under the influence of gravity, without taking the earth's kinetic energy into account.

If friction is negligible, the total energy is the sum of gravitational potential energy and the kinetic energy of the object. It is constant at different stages of the motion.

Example 5 A simple pendulum.



Example 6 Roller coaster ride.



The previous two equations are also valid for these situations.

$$mgh_a + \frac{1}{2}mv_a^2 = mgh_b + \frac{1}{2}mv_b^2 \qquad (1)$$

$$\text{or} \quad \Delta E_k + \Delta E_p = 0 \qquad (2)$$

In the roller coaster case, the total energy must be greater than *mgd*, where *d* (metres) is the diameter of the loop and zero potential energy is assigned to the lowest point, otherwise the carriage will be short of energy to reach the top of the loop.

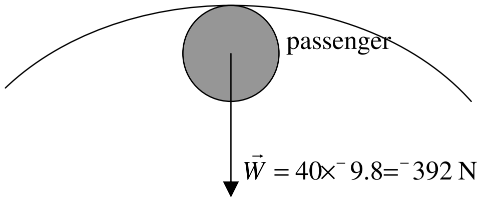
Example 7 A carriage moves under gravity at 8.5 m s^{-1} at the top of a circular roller coaster loop of radius 7.5 m.
(a) Calculate the speed of the carriage at the bottom of the loop if there is no resistive force.
(b) In another run the total mass of the carriage with more passengers in it is doubled. The speed at the top remains the same. What is its speed at the bottom?
(c) Calculate the reaction force on a 40 kg passenger at the top.
(d) Calculate the reaction force on this passenger at the bottom if there is a resistive force restricting the speed to 11 m s^{-1} at the bottom.

(a) $mgh_a + \frac{1}{2}mv_a^2 = mgh_b + \frac{1}{2}mv_b^2$, $\therefore 2gh_a + v_a^2 = 2gh_b + v_b^2$

$0 + v_a^2 = 2(9.8)(2\times 7.5) + 8.5^2$, $v_a \approx 19$ m s^{-1}

(b) The equation $2gh_a + v_a^2 = 2gh_b + v_b^2$ does not depend on the mass of the carriage, $\therefore$ same speed.

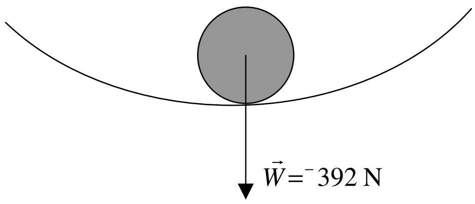
(c) Take upward as the positive direction. Let $\vec{R}$ (direction is unknown) be the reaction force on the passenger.



$\vec{F}_{net} = m\vec{a}$, $\vec{R} + {}^{-}392 = 40\times {}^{-}\left(\frac{8.5^2}{7.5}\right)$, $\vec{R} \approx {}^{+}7$ N.

The small upward reaction force is from the harness restraining the passenger from falling out of the seat.

(d)



$\vec{F}_{net} = m\vec{a}$, $\vec{R} + {}^{-}392 = 40\times {}^{+}\left(\frac{11^2}{7.5}\right)$, $\vec{R} \approx {}^{+}1000$ N

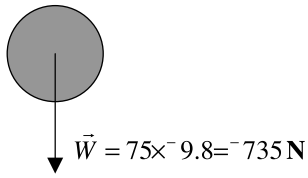
The large upward reaction force is from the seat providing the necessary force to keep the passenger in circular motion.

Example 8 During an air show a jet fighter plane makes a vertical circular manoeuvre. At the highest point of the loop (radius 1000 m) the plane flies at 99 m s^{-1} and the 75 kg pilot is in the upright position.
(a) Calculate the centripetal acceleration of the plane.
(b) What is the centripetal acceleration of the pilot?
(c) Determine the normal reaction force on the pilot.
(d) Is the total mechanical energy (kinetic energy + potential energy) conserve in the vertical circular manoeuvre?

(a) $a = \frac{v^2}{r} = \frac{99^2}{1000} = 9.8$ m s^{-2}

(b) 9.8 m s^{-2}

(c) Let $\vec{R}$ (direction is unknown) be the reaction force on the passenger.



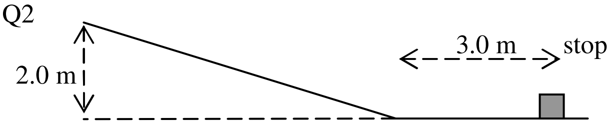
$\vec{F}_{net} = m\vec{a}$, $\vec{R} + {}^{-}735 = 75\times {}^{-}9.8$, $R \approx 0$ N.
The pilot feels weightless at the top of the loop.

(d) No, because there is additional energy from the jet fuel.

Questions: Next page

Q1 A 75 kg parcel is tied to a 10 m bungee cord which is fastened to a bridge. The cord has a force constant of 150 N m^{-1}. The parcel falls a vertical distance of 26 m to a stop when it is dropped from the bridge. (a) Calculate the change in gravitational potential energy of the parcel.
(b) Calculate the elastic potential energy in the bungee cord when the parcel is at its lowest point. (c) Determine the maximum kinetic energy of the parcel during the fall.

Q2



A 2.0-kg object slides down a slope from a height of 2.0 m. It starts from rest and comes to a stop on the horizontal plane after travelling 8.0 m in total. (a) Determine the amount of heat generated due to friction. (b) Determine the average force of friction against the motion of the object.
(c) Assume that the force of friction is constant throughout the motion; determine the maximum kinetic energy of the object.

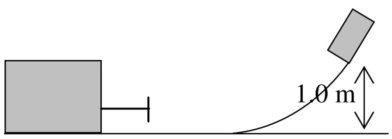
Q3 A 25 kg mass placed at the centre of a trampoline causes it to depress by 7.5 cm, whilst a 50 kg mass depresses the trampoline by 15 cm.
(a) Calculate the increase in elastic potential energy of the trampoline when the 25 kg mass is replaced by the 50 kg mass. (b) Calculate the maximum depression if the 50 kg mass is dropped from a height of 1.0 m above the centre of the trampoline.

Q4 A champagne cork popped out from a bottle at 1.2 m above the floor. It lands on the floor at a horizontal distance of 5.0 m from its starting point 2.0 s later. (a) Calculate the initial speed of the cork. (b) Use the conservation of energy idea to determine the maximum height reached by the cork. (c) Use the conservation of energy idea to determine the landing speed of the cork. (d) Comment on the landing speed of the cork if the bottle is tilted at a different angle and the initial speed of the cork remains the same.

Q5 A 45 kg skier skis over a frictionless circular hill of radius 15 m and then down the hill to a frictionless circular dip of 25 m radius. At the top of the hill the ground exerts an upward force of 320 N on the skier, and at the bottom of the dip the ground exerts an upward force of 1.1 kN on the skier. What is the speed of the skier (a) at the top of the hill (b) at the bottom of the dip?

Q6 A 10 g bullet strikes a 2 kg ballistic pendulum and remains embedded in it. The centre of mass of the pendulum with the bullet in it rises a vertical distance of 12 cm. Calculate the speed of the bullet just before it hits the pendulum.

Q7 A 2.0 kg steel block sliding down a frictionless curved plane from a height of 1.0 m to hit a nail of 5.0 g drives the nail 0.50 cm into a fixed wooden block without rebounding. Find the average force of resistance exerted by the wooden block.



Numerical, algebraic and worded answers: 1a. 19500 J 1b. 19200 J 1c. 9375 J 2a. 40 J 2b. 5 N 2c. 15 J 3a. 28 J 3b. 0.72 m 4a. 9.73 m s^{-1} 4b. 5.6 m approx. 4c. 10.9 m s^{-1} approx. 4d. Same 5a. 6.6 m s^{-1} approx. 5b. 19 m s^{-1} approx. 6. 311 m s^{-1} approx. 7. 3990 N approx. (Not 4000 N)