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Gravity

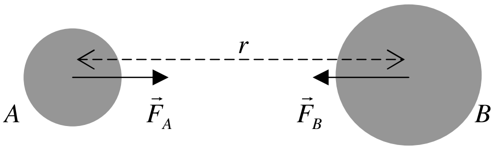
Newton's law of universal gravitation

For any two objects, A and B, there is always a mutual attractive force F between them. F has the following properties.

$$F \propto M_A,\ F \propto M_B,\ F \propto \frac{1}{r^2},\ \therefore F \propto \frac{M_A M_B}{r^2},\ \therefore F = \frac{GM_A M_B}{r^2}$$

M_A and M_B are the masses of A and B in kg respectively, r metres is the separation between the centres of mass of the objects and, G is the constant of proportionality known as the ***Universal constant****, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}*

The equation $F = \frac{GM_A M_B}{r^2}$ is called ***Newton's law of universal gravitation****.*



This force of attraction is mutual force, i.e. if B attracts A with force $\vec{F}_A$ then A attracts B with force $\vec{F}_B$, and $\vec{F}_A = -\vec{F}_B$ in accordance with Newton's third law and

$\left|\vec{F}_A\right| = \left|\vec{F}_B\right| = F = \frac{GM_A M_B}{r^2}$ according to Newton's law of universal gravitation.

Example 1 What is the average gravitational force between Earth (5.98×10^{24} kg) and the sun (1.99×10^{30} kg) with an average distance of 1.50×10^{11} m between them?

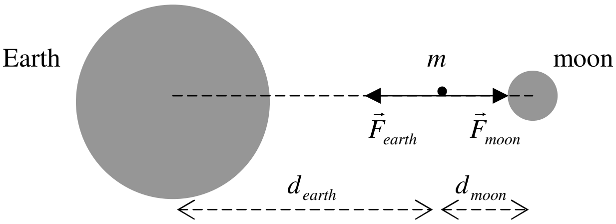
$$F = \frac{GM_A M_B}{r^2} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.99\times10^{30})}{(1.50\times10^{11})^2}$$

$= 3.53\times10^{22}$ N

Example 2 There is a point between Earth and the moon (7.36×10^{22} kg) where a spacecraft experiences zero net gravitational force due to these two bodies only. The average distance between Earth and the moon is 3.82×10^8 m.

(a) Find the value of the ratio $\frac{d_{earth}}{d_{moon}}$ at that point.

(b) How far is that point from Earth?



(a) $F_{moon} = F_{earth}$, $\frac{GM_{moon}m}{(d_{moon})^2} = \frac{GM_{earth}m}{(d_{earth})^2}$

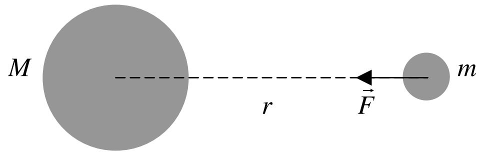
$$\frac{d_{earth}}{d_{moon}} = \sqrt{\frac{M_{earth}}{M_{moon}}} = \sqrt{\frac{5.98\times10^{24}}{7.36\times10^{22}}} \approx 9$$

(b) $d_{earth} : d_{moon} \approx 9:1$ $\therefore d_{earth} \approx \frac{9}{9+1}\times3.82\times10^8 \approx 3.4\times10^8$ m

Gravitational field of a planet

It is defined as the gravitational force on each kg of an object of mass m kg at some distance r from the planet of mass M kg, i.e. $\vec{g} = \frac{\vec{F}}{m}$.

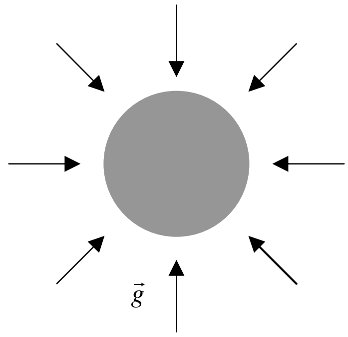
$\vec{g}$ is measured in N kg^{-1}.



Since $F = \frac{GMm}{r^2}$ $\therefore$ $g = \frac{F}{m} = \frac{GM}{r^2}$

$\vec{g}$ is in the same direction as $\vec{F}$, i.e. towards the centre of mass of the planet.

Note: M in the above equation is the mass of the planet. $\vec{g}$ of the planet is due to the mass of the planet and it is independent of the object m in the vicinity of the planet.



When an object m is placed in the gravitational field $\vec{g}$ of a planet, the force of gravity on it is $m\vec{g}$ which is also known as the weight $\vec{W}$ of the object, $\vec{W} = m\vec{g}$

Weight is a force and it is measured in newtons N. $\vec{W}$ and $\vec{g}$ are in the same direction, i.e. towards the centre of the planet.

Example 3 Determine the gravitational field of the moon and the weight of a 5 kg object at its surface, using only the values of *G*, the mass and the radius of the moon.

$$g = \frac{GM_{moon}}{r^2} = \frac{(6.67\times10^{-11})(7.36\times10^{22})}{(1.74\times10^6)^2} \approx 1.62 \text{ N kg}^{-1}.$$

$W = mg = 5\times1.62 \approx 8.1$ N.

Free fall and acceleration due to gravity

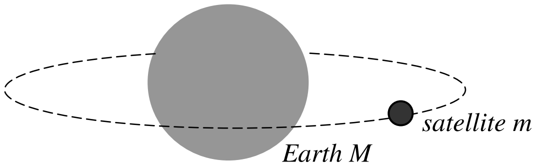
When an object moves in a gravitational field $\vec{g}$ and gravity is the ***only*** *force acting on it, its acceleration is, according to Newton's second law, $\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{m\vec{g}}{m} = \vec{g}$.*

Acceleration $\vec{a}$ is in the same direction as $\vec{g}$.

Motion under the influence of gravity ***only*** *is called* ***free fall*** *(not necessarily falling) and the acceleration is $\vec{g}$, e.g. orbiting satellites are in free fall.*

A firing rocket in flight is not in free fall because gravity is not the only force on the rocket. A sky diver is not in free fall because there is air resistance besides gravity.

Circular orbits under gravity



For a satellite in orbit around Earth, it is in free fall in Earth's gravitational field $\vec{g}$ *, its acceleration is* $\vec{a} = \vec{g}$ *, where*

$g = \frac{GM}{r^2}$.

The satellite is in uniform circular motion. Therefore, its acceleration is centripetal and $a = \frac{v^2}{r}$.

$\therefore \ \frac{v^2}{r} = \frac{GM}{r^2} \quad \therefore v^2 r = GM$

The last equation suggests that $v^2 r$ *is a constant for any satellite around Earth. The constant is GM, where M is the mass of Earth. Hence a satellite at a higher altitude has a lower speed.*

For any two satellites, A and B, orbiting around Earth,

$$v_A^2 r_A = v_B^2 r_B \ .$$

Kepler's third law

Centripetal acceleration of a satellite can also be $a = \frac{4\pi^2 r}{T^2}$ *where T is the period of revolution.*

$\therefore \ \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} \quad \therefore \frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

The last equation is known as ***Kepler's third law****, which shows the relationship between r and T.* $\frac{r^3}{T^2}$ *is a constant which depends on the mass M of the planet supplying the gravitational field. At a higher altitude the period of the satellite is longer. For any two satellites A and B orbiting around the same planet,*

$\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$.

Example 4 (2010 VCAA Exam 1) The international Space Station (ISS) is under construction in Earth orbit. It is incomplete, with a current mass of 3.04×10^5 kg. The ISS is in a circular orbit of 6.72×10^6 m from the centre of Earth. Given mass of Earth is 5.98×10^{24} kg, radius of Earth 6.37×10^6 m, and $G = 6.67\times10^{-11}$ N m^2 kg^{-2}.

(a) What is the weight of ISS in its orbit? (b) What is the period of orbit of the ISS around Earth? (c) When the ISS is completed in 2011, its mass will have increased to 3.70×10^5 kg. What will the period of its orbit be?

(a) $W = mg = m\frac{GM}{r^2} = (3.04\times10^5)\times\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.72\times10^6)^2}$

$= 2.69\times10^6$ N

(b) The ISS is in free fall, $a = g$, $\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$,

$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2(6.72\times10^6)^3}{(6.67\times10^{-11})(5.98\times10^{24})}} = 5.48\times10^3$ s

(c) The period will be the same because $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$ is independent of the mass of ISS.

Geostationary satellites

A geostationary satellite always appears to be at the same spot above the surface of Earth to an Earth bound observer. It is directly above the equator and has the same period of revolution as the rotation of Earth, i.e. a day. There is one and ***only one*** *orbit for all geostationary satellites.*

Example 5 Calculate the radius of the orbit of a geostationary satellite using the known values of *G* and the mass of Earth.

$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$, $r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(24\times60\times60)^2}{4\pi^2}}$

$\approx 4.23\times10^7$ m.

Planets in the solar system

The orbits of the planets in the solar system are elliptical, but can be approximated as circular to simplify calculations.

Example 6 Earth has an **average orbital radius** around the sun of 1.50×10^{11} m and its period of revolution is of course a year. Mars has an average orbital radius of 2.28×10^{11} m.
(a) Determine the period of Mars in Earth years using only the given data for Earth and Mars.
(b) Hence calculate the magnitude of Mars' acceleration.
(c) What is the gravitational field strength in N kg^{-1} of the sun at where Mars is?

(a) $\frac{r_A^3}{T_A^2} = \frac{r_B^3}{T_B^2}$, $\left(\frac{T_M}{T_e}\right)^2 = \left(\frac{r_M}{r_e}\right)^3$, $\frac{T_M}{T_e} = \left(\frac{r_M}{r_e}\right)^{\frac{3}{2}}$

$\therefore \ T_M = \left(\frac{r_M}{r_e}\right)^{\frac{3}{2}} T_e = \left(\frac{2.28\times10^{11}}{1.50\times10^{11}}\right)^{\frac{3}{2}} \times 1 = 1.87$ earth years

(b) $a = \frac{4\pi^2 r_M}{T_M{}^2} = \frac{4\pi^2(2.28\times10^{11})}{(1.87\times365\times24\times60\times60)^2} \approx 0.00258$ m s^{-2}

(c) $g = a \approx 0.00258$ N kg^{-1}.

Example 7 (2011 VCAA Exam 1) Assume that somewhere in space there is a small spherical planet with a radius of 30 km. By some chance a person living on this planet visits Earth. He finds that he weighs the same on Earth as he did on his home planet, even though Earth is so much larger. Refer to Example 4 for Earth data. $g \approx 10$ N kg^{-1} at the surface of Earth. (a) What is the value of the gravitational field on the surface of the visitor's planet? (b) What is the mass of the visitor's planet? (c) The visitor's home planet is in orbit around its own small star at a radius of orbit of 1.0×10^9 m. The star has a mass of 5.7×10^{25} kg. What would be the period of the orbit of the visitor's planet?

(a) *Weight = mass* $\times$ *g* , $g = \frac{weight}{mass}$, the same weight and the same mass, $\therefore$ *g* must be the same, $g \approx 10$ N kg^{-1}

(b) $g = \frac{GM}{r^2}$, $10 = \frac{(6.67\times10^{-11})M}{(30\times10^3)^2}$, $M = 1.35\times10^{20}$ kg

(c) $T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2(1.0\times10^9)^3}{(6.67\times10^{-11})(5.7\times10^{25})}} = 3.2\times10^6$ s

Questions: Next page

Q1 A 1 kg dumb-bell is at rest on the ground.
(a) Calculate the force of gravity exerted by the dumb-bell on Earth. (b) Calculate the net force exerted by the dumb-bell on Earth.

Q2 The centre of the moon is about 385000 km from the centre of Earth. The mass of the moon $= 7.36\times10^{22}$ kg and the mass of Earth $= 5.98\times10^{24}$ kg. Determine the gravitational field strength of the moon g_{moon} experienced by Earth.

Q3 Refer to the information in Q2. Determine the value of each ratio. (a) $\frac{g_{earth}}{g_{moon}}$ and (b) $\frac{F_{earth.on.moon}}{F_{moon.on.earth}}$,
where g_{earth} stands for gravitational field strength of Earth experienced by the moon, and F the magnitude of gravitational force.

Q4 There is a point between Earth and the moon where gravitational field due to both bodies is zero. Determine the distance of that point from the centre of the moon.

Q5 A satellite is in a stable circular orbit 250 km above the surface of Earth ($r_{earth} = 6380$ km). Calculate the orbital speed of the satellite.

Q6 Refer to Q5. Describe the effects on the motion of the satellite if the orbital speed is (a) reduced 'slightly', (b) increased 'slightly', and (c) increased 'greatly'.

Q7 Determine the orbital radius of a geostationary satellite.

Q8 The circular orbital speeds of satellite *A* and satellite *B* are v_A and v_B respectively, where $v_B = 2v_A$. Determine the value of each ratio. (a) $\frac{r_B}{r_A}$ and (b) $\frac{T_B}{T_A}$, where *r* stands for orbital radius, and *T* orbital period.

Numerical, algebraic and worded answers: 1a. 10 N 1b. 0 N 2. 3.3×10^{-5} N kg^{-1} 3a. 81 3b. 1 4. 3.8×10^{7} m 5. 7.76×10^{3} m s^{-1} 6a. The satellite will move into an elliptical orbit (closer to Earth). 6b. The satellite will move into an elliptical orbit (further from Earth). 6c. The satellite will move away from Earth in a hyperbolic path and never to return. 7. 4.2×10^{7} m 8a. 1/4 8b. 1/8