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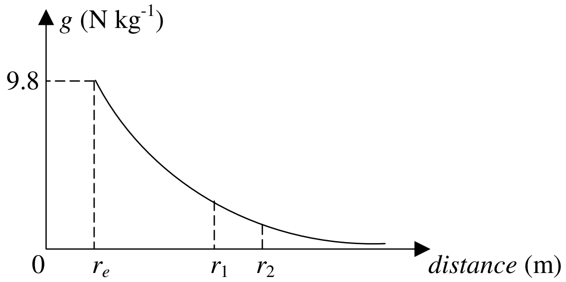
Gravitational field-distance graph (e.g. for Earth)



The distance is measured from the centre of Earth and r_e *is the radius of the earth.*

When an object moves from r_1 *to* r_2 *under gravity only, its gravitational potential energy increases while its kinetic energy decreases by the same amount. The change in energy for* ***each kilogram*** *of the object is given by the area under the graph from* r_1 *to* r_2*, and can be estimated from an accurately drawn graph.*

$$\textit{Change in energy} = \textit{mass of object} \times \textit{area under graph}$$

$$\Delta E = m \times \text{area under } g\text{-}x \text{ graph}$$

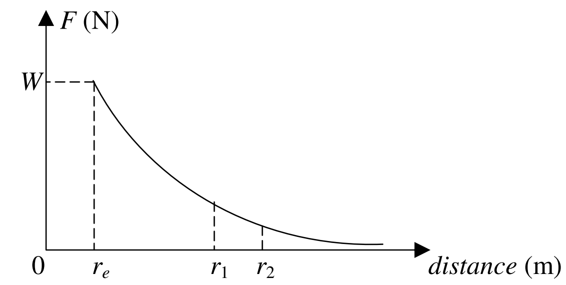
If it moves from r_2 *to* r_1*, potential energy decreases and kinetic energy increases.*

To carry a satellite from the surface of Earth to an orbit of radius r_1*, work (energy supplied to the satellite) is required and it is the area from* r_e *to* r_1 *for each kilogram of the satellite. Additional energy (kinetic) is required to make the satellite m to orbit around Earth M.*

Kinetic energy $E_k = \frac{1}{2}mv^2$ *and* $v^2 r = GM$,

$\therefore E_k = \frac{1}{2}m\left(\frac{GM}{r}\right) = \frac{GMm}{2r}$ *is the kinetic energy of an orbiting satellite.*

Note: If you are given a ***force of gravity-distance*** *graph, the area under the graph is the change in energy.*



W is the weight of the object at the surface of Earth.

Example 1 Given the values of *G* and the mass of Earth, calculate the kinetic energy of an earth satellite in circular orbit at an *altitude* of 3.60×10^4 km. The mass of the satellite is 350 kg and the radius of Earth is 6380 km.

Orbital radius $r = 6380 + 3.60 \times 10^4 = 4.238 \times 10^4$ km $= 4.238 \times 10^7$ m

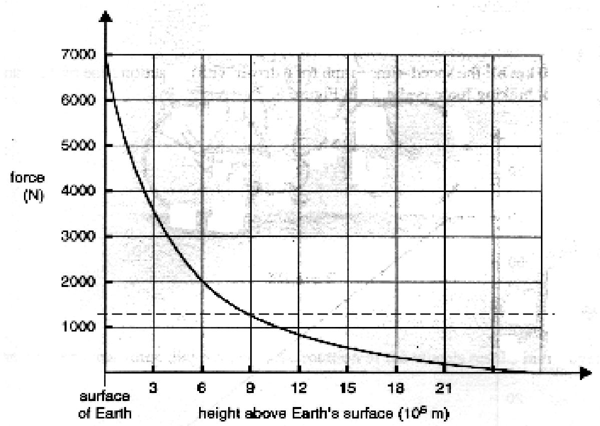
$$\therefore E_k = \frac{GMm}{2r} = \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})(350)}{2(4.238 \times 10^7)} = 1.65 \times 10^9 \text{ J}$$

Example 2 Estimate the total amount of energy required to bring the satellite in Example 1 from the surface of Earth to orbit around the planet.

From a suitable *g-x* graph, estimate the area under the graph from $x = 6.38 \times 10^6$ m to $x = 4.238 \times 10^7$ m, and multiply by 350 kg. This gives the energy required to take the 350 kg satellite to the specified altitude.

Estimated area $\approx 5.3 \times 10^7$ J kg^{-1}
Energy required to take the 350 kg satellite to the specified altitude $\approx (5.3 \times 10^7) \times 350 \approx 1.9 \times 10^{10}$ J
Total energy required to place the satellite in orbit around Earth $\approx 1.9 \times 10^{10} + 1.65 \times 10^9 \approx 2 \times 10^{10}$ J

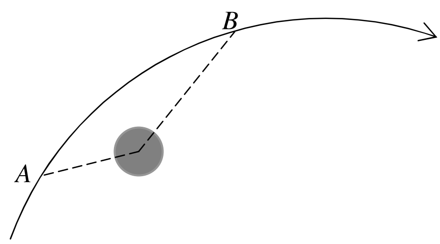
Example 3 (2002 VCAA Exam 2) The Mars Odyssey spacecraft was launched from Earth on 7 April 2001 and arrived at Mars on 23 Oct 2001. The graph shows the gravitational force acting on the 700 kg Mars Odyssey spacecraft against height above Earth's surface. Estimate the minimum launch energy needed for Mars Odyssey to escape Earth's gravitational attraction.



The dotted line shows the average gravitational force (1300 N approximately) acting on the 700 kg spacecraft from the Earth's surface to 26×10^6 m above Earth's surface.

The minimum launch energy needed for Mars Odyssey to escape Earth's gravitational attraction = total area under the graph $\approx 1300 \times 26 \times 10^6 \approx 3.4 \times 10^{10}$ J

Example 4 A piece of 250 kg space junk approaches Earth along an elliptical path from *A* to *B*. At *A* it has a speed of 4.0×10^4 m s^{-1}. At *B* its gravitational potential energy *increases* by 8.0×10^{10} J. Find its speed at *B*.



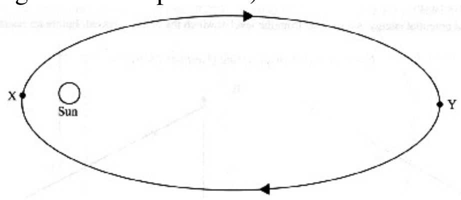
At *A*, $E_k = \frac{1}{2}(250)(4.0 \times 10^4)^2 = 2.0 \times 10^{11}$ J

At *B*, $E_k = 2.0 \times 10^{11} - 8.0 \times 10^{10} = 1.2 \times 10^{11}$ J

$\therefore \frac{1}{2}(250)v^2 = 1.2 \times 10^{11}$, $v \approx 3.1 \times 10^4$ m s^{-1}

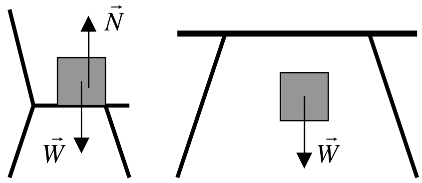
Example 5 (2008 VCAA Exam 1) The figure shows the orbit of a comet around the Sun. Describe how the speed and total energy of the comet vary as it moves in orbit from X to Y.

The comet moves at the highest speed at X (closest approach to the Sun) and decreases to the lowest speed at Y. The total energy (kinetic and gravitational potential) remains constant.



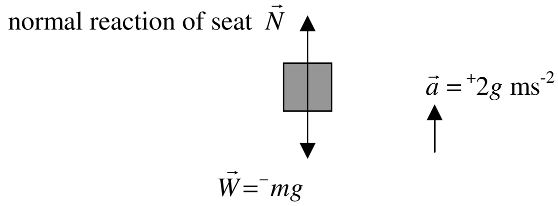
Weight and apparent weight

The weight $\vec{W}$ (measured in newtons) of an object of mass m (measured in kilograms) is the force of gravity on it and given by $\vec{W} = m\vec{g}$.
*When you sit on a chair, you can feel your weight because the chair exerts a normal reaction force on you. However, when you jump off a table you are in free fall, and there is no reaction force on you and you feel weightless. Motion under the influence of gravity **only** is called **free fall** (not necessarily falling).*



*The 'weight' that you feel is called your **apparent weight**. It is measured by the normal reaction force $\vec{N}$. When you are at rest on a chair, your apparent weight is the same as your weight. When you are in free fall, your apparent weight is zero.*

Example 6 At takeoff the acceleration of the rocket carrying the space shuttle is 2g upward (taken as the positive direction). Find the apparent weight of the $m$kg astronaut.



Apply Newton's second law to the astronaut: $\vec{F}_{net} = m\vec{a}$

$\vec{N} + \vec{W} = m\vec{a}$, $\vec{N} + {}^{-}mg = m({}^{+}2g)$, $\vec{N} = {}^{+}3mg$

The apparent weight is three times the actual weight mg. Therefore the astronaut feels three times as heavy during takeoff as before takeoff.

Example 7 What is the apparent weight of the astronaut while the space shuttle is in a circular orbit around the earth?

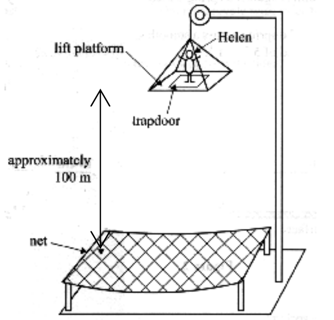
While the space shuttle is in orbit, it is in free fall. Inside the space shuttle the astronaut is also in free fall together with the space shuttle and experiences weightlessness, i.e. the apparent weight of the astronaut is zero.

Example 8 (2002 VCAA Exam 2) Refer to Example 3. While in deep space, on the way to Mars, Odyssey was travelling at a constant velocity of 23000 m s^{-1} and the spacecraft and all its contents were weightless. Explain why an object inside the spacecraft could be described as weightless.

This example is different from example 6. In deep space the spacecraft is far away from all objects and .: the total gravitational field is considered to be negligible.

Also the spacecraft was travelling at a constant velocity, .: there was no 'artificial' gravity. Hence an object inside the spacecraft could be described as weightless.

Example 9 (2009 VCAA Exam 1) Helen (60 kg) takes the ride on the platform to the top 100 m from the net.



(a) The platform travels vertically upward at a constant speed of 5.0 m s^{-1}. What is Helen's apparent weight as she travels up?
(b) As the platform approaches the top, it slows to a stop at a uniform rate of 2.0 m s^{-2}. What is Helen's apparent weight?
(c) Helen next drops through the trapdoor and falls to the net. Ignore air resistance. During her fall, Helen experiences 'apparent weightlessness'. Explain the last sentence.

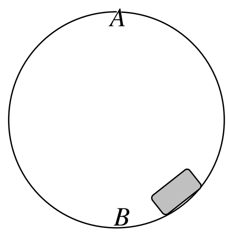
(a) Helen has no acceleration, net force is zero, .: $\vec{N} + m\vec{g} = \vec{0}$

${}^{+}N + 60 \times {}^{-}10 = 0$, $N = 600$.: *Helen's apparent weight* = 600 N

(b) $\vec{N} + m\vec{g} = m\vec{a}$, ${}^{+}N + 60 \times {}^{-}10 = 60 \times {}^{-}2.0$, $N = 480$ N
.: *Helen's apparent weight* = 480 N

(c) Helen is in free fall and there is no reaction force on her, .: she experiences 'apparent weightlessness'.

Example 10 (2011 VCAA Exam 1) A ride at a fun fair involves a car travelling around the rails inside a vertical circle of radius 15 m. Melanie (60 kg) is a passenger in the car.
(a) What is the minimum speed that the car can have at the highest point A if it is not to leave the rails and start to fall?
(b) What will be Melanie's apparent weight at the highest point A when the car is travelling at this minimum speed?
(c) As the car passes the lowest point B with a speed of 5.5 m s^{-1}, what is Melanie's apparent weight?



(a) At position A, $F_{net} = ma$, $mg + R = \frac{mv^2}{r}$.

The reaction force R on the car must be ≥ 0 for the car not to leave the rails. Let $R = 0$ to find the minimum speed.
.: $v = \sqrt{gr} = \sqrt{10 \times 15} = 12.2$ m s^{-1}.

(b) When the car (and Melanie) travels at the minimum speed at position A, both are in free fall and the reaction force of the car on Melanie is zero. Hence Melanie's apparent weight is zero at position A.

(c) At position B, $-mg + R = \frac{mv^2}{r}$.

$-600 + R = \frac{60 \times 5.5^2}{15}$, $R = 721$ N
Melanie's apparent weight is 720 N approximately.

Questions: Next page

Q1 A satellite is 36000 km above the surface of Earth. Estimate the increase in gravitational potential energy per kg of the satellite if its altitude is increased by 1 km.

Q2 (a) Estimate the gravitational potential energy gained when a 1405 kg weather satellite was lifted from Earth's surface to its circular polar orbit of radius 6.70×10^6 m.
(b) Calculate the kinetic energy of the satellite in its orbit.

Q3 Estimate the minimum energy needed to place a 650 kg geostationary satellite in orbit.

Q4 A 500 kg space probe is to be launched from Earth never to return. What initial kinetic energy will it need?

Q5 Describe the apparent weight of a 1 kg mass placed on the pan of a spring balance (a) when the pan is at rest
(b) when the pan is approaching the lowest point of its oscillation.

Q6 What is the apparent weight of a 65 kg person on a toboggan sliding down an icy slope inclined at $30°$ to the horizontal?

Q7 A stuntman driving a car over a circular hill top of radius of curvature 250 m experiences weightlessness at the top. What is the minimum speed needed for this experience?

Q8 Calculate the apparent weight of a 1 kg mass at rest on the moon's surface.

Numerical, algebraic and worded answers: 1. 222 J 2a. 4.2×10^9 J 2b. 4.2×10^{10} J 3. 3.8×10^{10} J 4. 2.5×10^{10} J 5a. The apparent weight has the same magnitude as its normal weight, i.e. 10 N. 5b. The apparent weight is greater in magnitude than the normal weight of the mass and reaches its maximum value. 6. 560 N 7. 50 m s^{-1} 8. 1.6 N